



Deliverable D4.2.1

Bayesian updating of PSHA models with instrumental seismicity data, introduction of correlation

WP	4	Hazard calculation and ground motion for engineering
Task	4.2.1	<i>Bayesian updating of PSHA models with instrumental seismicity data, introduction of correlation</i>
Ref.	SIGMA3-2025-WP4-4.2.1-02	

Dissemination level¹	PU	Deliverable Review Meeting (DRM) Date	
		Actual DRM Date	05/12/2025

Lead Partner	EDF
Contributing Partner(s)	

Document Version	Date	Author	Comments ²
01	03/10/25	Luis ALVAREZ SANCHEZ	
02	06/01/26	Luis ALVAREZ SANCHEZ	Reviewed version according to DRM comments

¹ Dissemination level: **PU** = Public, **RE** = Restricted, only for members of the SIGMA3 consortium

² Creation, modification, final version for evaluation, revised version following evaluation, final



Author(s) signature
Luis Guillermo ALVAREZ
SANCHEZ

ALVAREZ-
SANCHEZ
Luis-guillermo

Digitally signed by
ALVAREZ-SANCHEZ
Luis-guillermo
Date: 2026.09.02
17:00:03 +02'00'

Scientific Committee Chairman
signature


Dr.-Ing. P. Renault

Lead Partner signature

VIALLET
Emmanuel

Signature numérique
de VIALLET Emmanuel
Date : 2026.03.21
11:09:13 +01'00'

Deliverable Abstract

Probabilistic Seismic Hazard Analysis (PSHA) is the standard methodology for estimating the likelihood of exceeding ground-motion intensity thresholds at a given site, accounting for both aleatory and epistemic uncertainties. Despite its widespread adoption, PSHA remains subject to criticism due to the high uncertainty in its predictions and the limited availability of observational data for validation. This study proposes a novel framework for the direct evaluation and Bayesian updating of PSHA logic-tree weights using observed ground-motion records, explicitly addressing spatial correlation among multiple observation sites. The methodology combines event-based simulations of branch-consistent ground-motion fields, site-effect corrections, and operational-period modeling to ensure comparability with empirical data. Correlated exceedance counts are modeled using the Pólya (negative binomial) distribution, which captures variance inflation induced by spatial dependence, while joint multi-IML updating employs a Bernstein copula to represent dependence across intensity measure levels.

The framework is applied to the metropolitan France PSHA model (Drouet et al., 2020) using a reference database spanning 1977–2020 and comprising EDF industrial sites and EPOS-France stations. Results confirm the adequacy of the Pólya distribution for correlated exceedance modeling, with spatial correlation observed consistently at monthly resolution. EDF sites provide nearly the same effective observation time as the full network due to their wide spatial distribution, while EPOS-France stations contribute limited additional statistical power because of clustering. Bayesian updating improves model consistency, particularly when applied jointly across multiple IMLs, and highlights systematic overestimation at low thresholds. Sensitivity analyses reveal that uncertainties in station operability influence results but do not fully explain discrepancies between predicted and observed exceedances.

This correlation-aware, multi-IML evaluation framework enhances the statistical robustness of PSHA validation and provides a principled approach for data-informed logic-tree weighting, offering clear pathways for methodological refinement and regulatory application.

Action 4.2.1 abstract

This research action advances probabilistic seismic hazard computation by introducing a methodology that strengthens the evaluation of PSHA logic trees through explicit treatment of spatial correlations among observation sites. Traditional approaches assume site independence, often requiring the exclusion of correlated stations and thus limiting the use of available data. By incorporating correlations via the Polya distribution, the methodology maximizes data usage and improves the robustness of model evaluations.

The approach also accounts for the actual operational history of observation sites, reducing biases from assuming continuous operation. Furthermore, it employs multivariate distributions to update logic tree weights across multiple Intensity Measure Levels (IMLs) within a Bayesian framework, offering a statistically rigorous means of refining model credibility.

Applications to PSHA models for metropolitan France confirm the utility of the methodology, showing that long observation periods and low-correlation site configurations enhance statistical reliability. Overall, this work provides a practical framework for more realistic, comprehensive, and data-efficient PSHA evaluations, directly contributing to improved confidence in seismic hazard assessments under real observational constraints.

Table of Content

1. Introduction.....	7
2. Evaluation and updating of PSHA models through Bayesian Inference.....	10
2.1. Distribution of the total number of exceedances for independent sites	10
2.2. Distribution of the total number of exceedances for dependent sites.....	11
3. Case study – Evaluation of a PSHA model for metropolitan France: Drouet et al., (2020).13	
3.1. Logic-tree model	13
3.2. Observation sites and reference database of observed ground motions	13
3.3. Estimation of the correlated distributions of exceedances predicted by the logic tree branches.....	15
3.3.1. Simulation of branch consistent distributions of correlated exceedances.....	15
3.3.2. Modeling of site effects.....	16
3.3.3. Definition and assessment of station operational time frames	17
3.3.4. Construction of the correlated distribution of exceedances for evaluation of logic tree branches	18
3.4. Evaluation for a single IML.....	22
3.5. Generalized evaluation for multi IMLs	25
4. Discussion	28
5. Conclusion and perspectives	33
5.1. Limitations.....	34
5.2. Perspectives	34
6. Bibliography.....	35
7. Annex	38
7.1. Site effects terms for observation sites considered in this study.....	38
7.2. Goodness-of-fit of the Pólya distribution to the cumulative number of exceedances ..	39
7.3. Drouet et al., 2020 evaluation considering EDF observation sites.....	41

Glossary

A	Mean of the Pólya distribution
EAU	Réseau d'Alerte Sismique (Seismic Alert Network)
$EC8$	Eurocode 8
EDF	Electricité de France
GIT	Generalized Inversion Technique
GMM	Ground Motion Model
IM	Intensity Measure
IML	Intensity Measure Level
K	Correlation parameter of the Pólya distribution
k_{SSI}	Coefficient for Soil-Structure Interaction transformation
K_{soil}	Coefficient for site to reference site transformation
M_w	Moment magnitude
$N_{catalogues}$	Number of catalogues of simulated ground motion fields
N_{obs}	Number of times a given IML has been exceeded
$N_s(t)$	The number of exceedances of a given IML within time " t " at site " s "
$N_{tot}(t)$	The number of exceedances of a given IML within time " t " at an ensemble of sites of interest
OQ	OpenQuake engine
$P(Branch_i)$	The prior for branch " i ", is the probability originally assigned to branch " i " in the logic tree model
$P(Branch_i N = N_{obs})$	The posterior for branch " i ", represents the probability of the model of the branch " i " to be the good model to describe the observed number of exceedances of the IML of interest
$P(N=N_{obs} Branch_i)$	The likelihood of the observation, N_{obs} , for the distribution of branch " i ".
PGA	Peak Ground Acceleration
PGA_{ref}	Simulated Peak Ground Acceleration according to the models defining a logic tree branch and for the site reference conditions considered in the PSHA model
PGA_{site}	Simulated Peak Ground Acceleration according to the models defining a logic tree branch and including site effects I
PGA_{RotD50}	Peak Ground Acceleration, Rotated to the Median of Directions (50th percentile).
$PSHA$	Probabilistic Seismic Hazard Analysis
EPOS-FRANCE	Réseau Sismologique et Géodésique Français
$R_{hypocentral}$	Hypocentral distance
S	Number of sites considered in the computation of the number of exceedances
SA	Spectral acceleration
SSI	Soil-Structure Interaction
T	Number of years of seismicity simulated with the OQ engine
t	Investigated period of time in a PSHA computation
t_i	Period of time for which the Pólya distribution is to be scaled
V_{s30}	Average Shear-Wave velocity of the upper 30 meters
λ_s	Rate of exceedance of a certain IML for site " s "
Λ	The cumulated rate of exceedance of the set of sites considered in the analysis

τ	Standard deviation of the inter-event residuals
Φ	Standard deviation of the intra-event residuals
Φ_{S2S}	Standard deviation of the site-to-site component of the intra-event residuals
Φ_{SS}	Standard deviation of the single station component of the intra-event residuals
r	Degree of over-dispersion of the Polya distribution
p	Probability of success in Bernoulli trials
S_{ess}	Effective number of observation sites in a correlated count context
T_{ess}	Effective independent observation time in a correlated count context
ϵ_{intra}	Intra-event residual
N_{wi}	Cumulative number of exceedances counts for a given IML predicted by the logic-tree model using the initial (prior) set of branch weights.
$N_{wf(IML)}$	Cumulative number of exceedances counts for a given IML predicted by the logic-tree model using the updated (posterior) set of branch weights for a given IML.
$N_{wf(joint)}$	Cumulative number of exceedances counts for a given IML predicted by the logic-tree model using the updated (posterior) set of branch weights for a joint set of IMLs.
A	Mean of the Polya distribution
<i>EAU</i>	Réseau d'Alerte Sismique (Seismic Alert Network)
<i>EC8</i>	Eurocode 8
<i>EDF</i>	Electricité de France
<i>GIT</i>	Generalized Inversion Technique
<i>GMM</i>	Ground Motion Model
<i>IM</i>	Intensity Measure
<i>IML</i>	Intensity Measure Level
K	Correlation parameter of the Polya distribution
k_{SSI}	Coefficient for Soil-Structure Interaction transformation
K_{soil}	Coefficient for site to reference site transformation
M_w	Moment magnitude
$N_{catalogues}$	Number of catalogues of simulated ground motion fields
N_{obs}	Number of times a given IML has been exceeded
$N_s(t)$	The number of exceedances of a given IML within time “ t ” at site “ s ”
$N_{tot}(t)$	The number of exceedances of a given IML within time “ t ” at an ensemble of sites of interest
<i>OQ</i>	OpenQuake (engine)
$P(Branch_i)$	The prior for branch “ i ”, is the probability originally assigned to branch “ i ” in the logic tree model
$P(Branch_i N = N_{obs})$	The posterior for branch “ i ”, represents the probability of the model of the branch “ i ” to be the good model to describe the observed number of exceedances of the IML of interest
$P(N=N_{obs} Branch_i)$	The likelihood of the observation, N_{obs} , for the distribution of branch “ i ”.
<i>PGA</i>	Peak Ground Acceleration

PGA_{ref}	Simulated Peak Ground Acceleration according to the models defining a logic tree branch and for the site reference conditions considered in the PSHA model
PGA_{site}	Simulated Peak Ground Acceleration according to the models defining a logic tree branch and including site effects I
$PSHA$	Probabilistic Seismic Hazard Analysis
EPOS-FRANCE	Réseau Sismologique et Géodésique Français
$R_{hypocentral}$	Hypocentral distance
S	Number of sites considered in the computation of the number of exceedances
SA	Spectral acceleration
SSI	Soil-Structure Interaction
T	Number of years of seismicity simulated with the OQ engine
t_1	Period of time for which the Polya distribution is to be scaled
V_{s30}	Average Shear-Wave velocity of the upper 30 meters
λ_s	Rate of exceedance of a certain IML for site "s"

1. Introduction

Probabilistic Seismic Hazard Analysis (PSHA), as first introduced by Cornell (1968), is a method used to estimate the probability of exceeding specific thresholds of an Intensity Measure (IM) at a given location, due to the cumulative effect of all potential seismic sources nearby. The outcome of PSHA is a probability distribution representing the exceedance probability of different IM thresholds, or Intensity Measure Levels (IMLs), over a defined period. Common IMs include Peak Ground Acceleration (PGA), Spectral Acceleration (SA) at specific periods (or frequencies) of vibration, etc. The aggregation of the probabilities of exceedance across various IMLs forms the hazard curve. PSHA incorporates uncertainties related to the size, location, and frequency of occurrence of earthquakes, as well as the conditions of the site of interest itself. Epistemic uncertainties are usually incorporated using a logic tree model. This model captures epistemic uncertainty by considering multiple hypotheses such as for seismic sources and ground motion prediction models, each assigned a weight based on expert judgment, or by any other type of data-informed methodology. Aleatory uncertainty, which refers to the inherent randomness in natural processes, is also a crucial component in PSHA. It is considered by incorporating the natural variability of earthquake occurrences and ground motion characteristics into the statistical models. This is typically done through the use of probability density functions that describe the distribution of possible values for different parameters, such as earthquake magnitude, location, and ground motion intensity. For example, Ground Motion Models (GMMs) are used to estimate the expected ground motion at a site given an earthquake of a certain magnitude and distance, including terms that account for the aleatory variability in ground motion.

Regardless of its widespread use and the fact that this analysis is today considered as standard practice in seismic hazard assessment, PSHA is often questioned due to the high levels of uncertainty in its estimates (Zanetti, Chiffi, & Petrini, 2023), such as for example the location and temporal occurrence of future events (uncertainties of the source), or even more, uncertainties on GMMs. Moreover, PSHA is often questioned on how epistemic and aleatory uncertainties influence the selection of seismic hazard models and their results (including the necessity of well separating aleatory uncertainties from epistemic ones, considering that their impacts on PSHA results are different). Therefore, the evaluation of PSHA models is crucial for ensuring their accuracy and reliability of its estimates (International Atomic Energy Agency, 2024).

The concept of PSHA evaluation is relatively new with respect to PSHA itself (Organisation for Economic Co-Operation and Development (OECD) / Nuclear Energy Agency, 2015), mainly because global earthquake ground motion databases are scarce, even more to a regional scale. The most rational and straightforward way to validate these distributions is the comparison of the predicted probabilities of exceedance of a certain IML, with statistics (counts) obtained from recorded data. This direct comparison, as defined in Weatherill et al., (2023), allows not only to draw conclusions on the representativity of each branch of the logic tree model with respect to the observed data, but also to update the weight assigned to each of these by means of techniques such as Bayesian updating, e.g., Secanell & Drouet (2021), Humbert (2015). According to studies such as Weatherill (2023), this type of comparison where models are contrasted to observed data are typically referred to as “direct”, whereas comparisons with respect to inferences made from observations such as macro seismic intensity are referred to as “indirect”. One of the first applications of direct evaluation is attributed to Ordaz et al., (1999), where the authors compared exceedance rates of PSHA models and observations at a single site in Mexico.

Despite the straightforwardness of the direct comparison method, the lack of data makes it almost impossible to evaluate the PSHA estimates at a single site. Authors such as Beauval et al (2008), Mak et al., (2014) and Weatherill et al., (2023) have concluded that earthquake processes, operating on time scales in the order of thousands of years cannot necessarily be validated over a few decades of observation (most often interrupted than and scarce than the opposite). To address this challenge, research by Beauval et al. (2008), Stirling et al., (2006), Fujiwara et al. (2009), and Albarello & D'Amico (2008) have pooled observations from multiple sites. This strategy increases the statistical power of the ground motion data. However, it necessitates meticulous consideration of the spatial correlation structure that appears between the ground motion observed at different sites, a feature not considered in single-site PSHA.

The assumption of independence is fundamental to standard PSHA, which models earthquake occurrence as a Poisson process. Under this framework, the probability of a future earthquake is considered independent of past events, and seismic hazard estimates at different sites are treated as mutually independent. However, this assumption represents a simplification of the complex nature of seismicity and is strictly valid only when considering a single site and mainshock PSHA (no aftershocks or foreshocks considered). When addressing multiple sites, the exceedance of an IML at one site is often dependent on exceedance at nearby sites, particularly when both are affected by the same earthquake. These dependencies arise from the spatial correlation of ground motions, which probabilistic models capture through correlated GMM intra-event residuals (ϕ). Consequently, the aggregate of ground-motion exceedances across a region with multiple observation sites does not follow a Poisson distribution but rather corresponds to the sum of correlated Poisson processes. In this study, inter-event residuals (τ) between different earthquakes are assumed to be uncorrelated

The effect of spatial correlation of the intra-event residuals in ground motion, which decreases with distance (Jayaram & Baker, 2009), is typically limited in PSHA evaluation applications by comparing to data observation sites situated beyond a predetermined minimum distance, e.g., Secanell et al., (2021). While this method effectively excludes correlated observations and allows for coherent comparison of exceedance probabilities computed from PSHA models with observed data across multiple sites, it faces several challenges. These include the somewhat subjective selection of the minimum distance between stations and the exclusion of valuable data by rejecting already scarce observation sites. This is particularly problematic in regions with clustered seismicity, where recording stations are densely arrayed. To address these challenges, contemporary research has begun to explore the implications of spatial dependence. For example, Iervolino et al. (2021) investigated the effects of spatial dependence on hazard evaluations within a broader multi-hazard framework. Additionally, Albarello & Peruzza (2017) suggested modifications to the original methodology developed by Albarello & D'Amico (2008) to incorporate the effects of spatial correlation.

More recently, different studies have proposed different non-Poissonian models to consider of the spatial correlation between observations at different sites and thus increase the data available for the evaluation of PSHA models. Such is the case of the Pólya distribution, as proposed in Humbert (2015), which not only has been proved to satisfactorily model this overdispersion produced by the spatial correlation at different observation sites, but also to provide a simple transition between the independent site model (where only one site is concerned and modelled by the Poisson distribution), and the more generic multi-site case.

With this context in mind, the objective of this study is to propose a methodology for implementing the Pólya distribution as a tool to model intra-event spatial correlation observed

in recorded data, thereby improving the evaluation of PSHA models by leveraging all available observation sites and increasing the statistical robustness of the assessment. The first section introduces the methodology for PSHA model evaluation using a Bayesian updating framework, including formulations for both independent and correlated observation sites. For independent sites, the Poisson distribution is employed, whereas for correlated sites, the Pólya distribution is proposed. The subsequent section illustrates the application of this methodology to the evaluation of the latest PSHA model for metropolitan France (Drouet, Ameri, Le Dortz, Secanell, & Senfaute, 2020), using a network of observation sites from industrial and civil operators and a reference database of observed ground motions compiled for the evaluation period 1977–2020. Details regarding the numerical estimation of the distribution of correlated exceedances used in the evaluation are provided in dedicated sections. Finally, the paper concludes with a discussion of key findings and perspectives for future research.

2. Evaluation and updating of PSHA models through Bayesian Inference

This study proposes a methodology to evaluate and update PSHA logic tree weights using a Bayesian framework. Within this approach, branch weights in the PSHA logic tree are revised according to their likelihood of reproducing the observed number of exceedances of a given IML in a reference database of recorded ground motions (N_{obs}). This revision process corresponds to Bayesian updating, where prior weights are adjusted based on observed evidence. Equation (1) provides the mathematical formulation for this updating procedure.

$$P(Branch_i | N = N_{obs}) = \frac{P(N = N_{obs} | Branch_i)P(Branch_i)}{\sum_{i=1}^{n_{branches}} P(N = N_{obs} | Branch_i)P(Branch_i)} \quad (1)$$

Where $P(Branch_i | N = N_{obs})$, often referred to as **the posterior**, represents the likelihood of the model of the branch i to be the right model to describe N_{obs} , i.e., the updated weight of the branch. $P(Branch_i)$, often referred to as **the prior**, is the likelihood originally assigned to branch i in the logic tree model, i.e., the initial weight of the branch. $P(N = N_{obs} | Branch_i)$ is the **likelihood of the observation**, N_{obs} , for the distribution of branch i .

2.1. Distribution of the total number of exceedances for independent sites

The probability of exceedance at each IML in the hazard curve is calculated using a statistical distribution that models the uncertainty associated with ground motion as modeled for each branch i . Traditional Probabilistic PSHA considers the Poisson distribution, which assumes independence between the probabilities of exceedance of a given IML at different sites and over a period of time t .

In scenarios where the temporal distribution of seismic events follows a Poisson process, the random variable $N_s(t)$, which counts the number of exceedances of a given IML within time t at site s , follows a Poisson distribution parameterized by the yearly rate of occurrence, λ_s . This parameter represents the expected annual frequency of seismic events that exceed a given IML at site s . The probability of observing n exceedances within time t can then be expressed as follows:

$$\mathbb{P}[N_s(t) = n] = \frac{e^{-\lambda_s t} (\lambda_s t)^n}{n!} \quad (2)$$

The hazard at a given IML, at site s , is constructed by computing the probability that there is at least one exceedance of the IML considered. That is $\mathbb{P}[N_s(t) \geq 1]$, which can be rewritten as:

$$\mathbb{P}[N_s(t) \geq 1] = 1 - \mathbb{P}[N_s(t) = 0] = 1 - e^{-\lambda_s t} \quad (3)$$

When considering S independent sites, the total number of exceedances $N_{tot}(t)$ over these sites during the period of time t is the sum of the independent Poisson distributions describing each site. Consequently, $N_{tot}(t)$ still follows a Poisson distribution, with its rate being the sum of the rates of each individual Poisson distribution.

$$N_{tot}(t) = \sum_{s=1}^S N_s(t) \quad (4)$$

$$\mathbb{P}[N_{tot}(t) = n] = \frac{e^{-\tau t} (\tau t)^n}{n!} \quad (5)$$

$$\Lambda = \sum_{s=1}^S \lambda_s \quad (6)$$

Where Λ is the cumulated rate of exceedance of the ensemble of S considered sites. The formulation presented in equation (6) considers that all sites have the same period of observation t . If this is not the case, the probability of exceedance for the ensemble of sites is computed according to equations (7) and (8).

$$\mathbb{P}[N_{tot}(t) = n] = \frac{e^{-\tau} (\tau)^n}{n!} \quad (7)$$

$$\Lambda = \sum_{s=1}^S \lambda_s t_s \quad (8)$$

Where, t_s is the observation time of site s .

2.2. Distribution of the total number of exceedances for dependent sites

When analyzing exceedance counts across multiple (nearby) observation sites, the assumption that the total number of exceedances $N_{tot}(t)$ follows a Poisson distribution becomes invalid due to inter-site dependencies. A single earthquake may be recorded by several stations, introducing correlation that inflates variance beyond the Poisson expectation (where variance equals the mean). This phenomenon, known as overdispersion, requires a more flexible probabilistic model.

To capture this behavior, this study adopts the **Pólya distribution** (negative binomial), following Humbert (2015) and leveraging its established use in other scientific domains such as biodiversity (Stoklosa, Blakey, & Hui, 2022) and population genetics (Gorczak, Burzykowski, & Claesen, 2025), among others. The Pólya distribution is particularly well-suited for correlated seismic networks because its parameterization allows the variance to grow faster than the mean when required, a property that enables modeling the additional uncertainty introduced by spatial correlation.

Formally, the Pólya distribution is parametrized by $r > 0$, a positive number that control the degree of overdispersion relative to a Poisson distribution, and $p \in (0,1)$, a probability of success in the Bernoulli trials interpretations. The probability mass function for the correlated number of counts, N_{tot} over a period t is given in equation (9), while equations (10) and (11) give the mean and the variance of the distribution, respectively.

$$\mathbb{P}[N_{tot}(t) = N] = \mathcal{P}(r, p, N) = \frac{\Gamma(N+r)}{N! \Gamma(r)} p^r (1-p)^N, \forall N \in \mathbb{N} \quad (9)$$

$$\mathbb{E}[N_{tot}(t)] = \frac{r(1-p)}{p} = A \quad (10)$$

$$\text{Var}[N_{tot}(T)] = A + \frac{A^2}{r} \quad (11)$$

In the adopted notation, the mean of the distribution is presented by A . From equations (9) and (10), it follows that the variance grows at a faster rate than the mean, primarily due to the additional term A/r^2 , which accurately reflects the effect of the correlation. To quantify this, this study introduces a correlation parameter defined as:

$$K = \frac{\text{Var}[N_{tot}(T)]}{\mathbb{E}[N_{tot}(t)]} \quad (12)$$

Under independence, $K=1$ (Poisson case), while under perfect correlation $K=S$, where S is the number of correlated observation sites. For partial correlation:

$$K = 1 + \frac{\sum_{1 \leq i \neq j \leq S} \text{Cov}(N_i(t), N_j(t))}{\mathbb{E}[N_{tot}(t)]} \quad (13)$$

It is important to emphasize that the Pólya distribution is used here to model inter-site dependencies, not the occurrence of earthquakes, which this study considers following a Poisson process, as is standard practice for main-shock PSHA. This assumption ensures that the yearly number of exceedances of a given IML (modeled through the previously described Pólya distribution) remains independent from one year to another. Consequently, this allows for the scaling of the distribution, obtained from data over a given number of years t , to a desired number of years t_1 .

$$N_{tot}(t_1) \sim \mathcal{P}\left(\frac{t_1}{t} A, K\right) \quad (14)$$

While the Pólya distribution captures the statistical effect of correlation through the parameter K , it is important to recognize its practical implication: correlation reduces the effective contribution of additional stations. In other words, adding more sites does not proportionally increase independent information because multiple stations often record the same events. This redundancy inflates variance without a corresponding increase in unique observations, which is precisely reflected in the accelerated growth of variance relative to the mean in equation (11).

To account for this, we introduce the concept of the effective number of independent stations (S_{ess}), defined as:

$$S_{ess}(t_1) = \frac{S}{K} \quad (15)$$

This metric adjusts the nominal station count by the correlation factor K , providing a realistic measure of informational gain. Assuming all stations S are operational in the analyzed period of time t_1 , the corresponding effective observation time (T_{ess}) becomes:

$$T_{ess} = \frac{S t_1}{K} \quad (16)$$

3. Case study – Evaluation of a PSHA model for metropolitan France: Drouet et al., (2020)

To illustrate the applicability of the Pólya distribution for modeling correlated exceedance counts -and, by extension, for assessing PSHA models by explicitly modelling the spatial correlation between observation sites – the following sections of this study present its implementation in the evaluation of the PSHA model for metropolitan France as proposed by Drouet et al. (2020) and as implemented in the OpenQuake (OQ) engine (Pagani, et al., 2014) by Weatherill et al., (2022), with supporting datasets provided in Weatherill (2022). The evaluation presented herein covers the period 1977–2020, selected to align with the temporal coverage of the reference databases for the sites considered (see next section). However, a broader evaluation period could be adopted in the future, depending on the availability and completeness of additional data.

3.1. Logic-tree model

The PSHA model was developed to predict $PGARotD_{50}$ for a reference site condition of $V_{s30} = 800$ m/s (mean shear wave velocity of the uppermost 30m). The model comprises 1,600 branches organized across four hierarchical levels. At the top level, the branches are grouped into four primary source model families: EDF, IRSN, GTR, and ZNL (zoneless). Within each family, 100 source models are considered (second level), with each model representing a single branch that captures the uncertainty in the seismic activity parameters and maximum magnitude, while the zonation (i.e., limits of the defined area sources or kernels for the ZNL model) remain consistent across all models within the family.

Each source model branch is then combined with four ground motion models (third level)—A17 (Ameri, Drouet, Traversa, Bindi, & Cotton, 2017), ASK14 (Abrahamson, Silva, & Kamai, 2014), DC15 (Drouet & Cotton, 2015), and C15 (Cauzzi, Faccioli, Vanini, & Bianchini, 2015).—producing four branches per source model (fourth level). This hierarchical combination results in 400 branches per source model family, and across the four families, the complete PSHA model is represented by 1,600 branches.

3.2. Observation sites and reference database of observed ground motions

The evaluation was conducted using observed ground motions recorded across a network of observation sites. The network comprises 100 sites: 19 EDF operational facilities and 81 stations from the civilian French EPOS network. **Figure 1** shows the spatial distribution of these stations across mainland France.

For this study, the total number of stations was limited to 100 to reduce computational demands while preserving representativity. Priority was given to EDF sites, as they provide the longest observation periods and the most reliable operation, being essential for plant exploitation. The EDF network includes Fessenheim, Bugey, Tricastin, Gravelines, Dampierre, Blayais, Saint Laurent, Chinon, Paluel, Saint Alban, Flamanville, Belleville, Nogent, Cattenom, Penly, Golfech, Chooz, Civaux, and Cruas. Each EDF site is equipped with accelerometers from the Seismic Alert Network (EAU), installed at the base of reactor buildings and triggered when ground acceleration exceeds a preset threshold of $PGA=0.01$ g in any component. The

remaining stations were selected from EPOS-France, favoring those with the longest installation periods to ensure consistency and data quality.

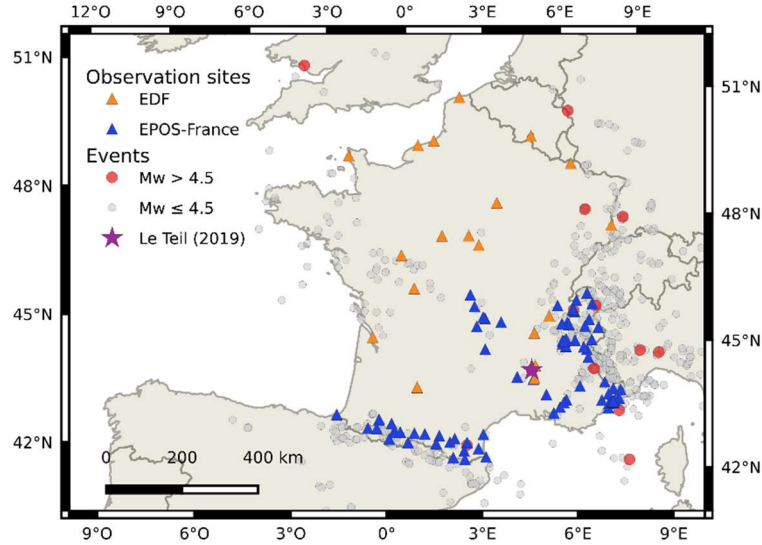


Figure 1. Spatial distribution of the observation sites and earthquakes considered in construction of the reference database of observed ground motions. In this figure, the observation sites are color-marked according to their operator, whereas the earthquakes are differentiated by the minimum magnitude and declustering criteria of the evaluated PSHA model. The Le Teil earthquake of 2019 is highlighted for reference purposes.

To generate the database of observed reference ground motions, two complementary sources were integrated: (i) a subset of the updated EPOS-France Earthquake Ground Motion Database for mainland France (1996–2021), as reported by Buscetti et al. (2024), and (ii) recordings from the EAU network monitoring EDF’s operational sites. Consistent with the minimum magnitude criterion of the PSHA model under scrutiny, the database was restricted to seismic events with a moment magnitude of 4.5 or greater ($M_w \geq 4.5$). Furthermore, only data from mainshock events were retained. For consistency, these events were identified using the Gardner-Knopoff declustering algorithm (Gardner & Knopoff, 1974), with parameters specified in Drouet et al., (2020). **Figure 1** shows the epicenter of the events recorded by the considered network during the evaluation period (1977-2021).

Figure 2a and **Figure 2b** are based on the dataset obtained after the complete filtering process—which includes both declustering and minimum magnitude filtering—reducing the number of records from 18,483 to 453. **Figure 2a** displays the distribution of the causal parameters, namely moment magnitude (M_w) and hypocentral distance ($R_{hypocentral}$), for the retained seismic records. **Figure 2b** illustrates the attenuation of the IM with respect to $R_{hypocentral}$ for the same set of recordings. In these figures, the grey points represent filtered seismic events; the purple points correspond to observations related to the Le Teil earthquake; and the remaining-colored points (in blue) indicate observations associated with other mainshocks.

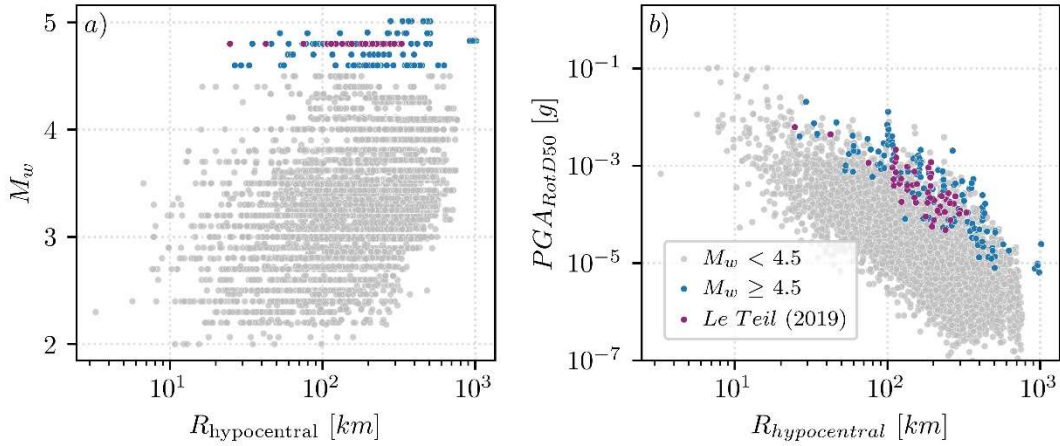


Figure 2. (a) - Distribution of the causal parameters of the events populating the database of observed reference ground motions, and (b) – Distribution of the PGA_{RotD50} of the records populating the reference database as a function of hypocentral distance. The data related to the Le Teil earthquake of 2019 ($M_w=4.9$) is highlighted in the figures for reference, whereas the data discarded because of the filtering criteria is shown in grey.

3.3. Estimation of the correlated distributions of exceedances predicted by the logic tree branches

To evaluate the PSHA model, it is necessary to compute the correlated distributions of exceedances predicted by each branch of the logic tree for the observation sites represented in the reference ground-motion database. To achieve this, this study proposes the use of site-specific synthetic ground-motion fields consistent with the source and ground-motion models defined in each logic tree branch.

The following sections describe the generation of these fields and their application in estimating the distributions, which are subsequently compared with the reference observed ground-motion database using the method outlined in Section 2. For brevity, intermediate results are presented only for the set of (100) branches corresponding to the EDF source model combined with the A17 ground-motion model.

3.3.1. Simulation of branch consistent distributions of correlated exceedances

To derive the correlated distributions, the first step is to generate ground-motion fields that incorporate spatial correlation among observation sites. This study employs the *event-based* functionality of the OQ engine to simulate correlated ground-motion fields according to the source and ground-motion models of a given logic-tree branch. Simulations are performed for a reference site condition of $V_{s30}=800$ m/s and for a specified number of realizations (catalogs) over a given time span (in years). The OQ framework simulates spatially correlated ground motions by sampling inter-event and intra-event residuals within ground-motion models. Inter-event residuals are assumed to be uncorrelated between earthquakes but constant across all sites affected by the same event. Intra-event residuals, on the other hand, are modeled using the spatial correlation approach proposed by Jayaram and Baker (2009), where correlation depends on both distance and spectral period.

Because site-specific effects are subsequently incorporated, a correction to the simulated ground motions is required. The original synthetic ground motion fields were sampled assuming full intra-event variability (ϕ), i.e., under an ergodic assumption. Adding site effects without adjustment of the uncertainty level would result in double-counting the site-to-site variability component (ϕ_{SS}) of the ground-motion model (see Al Atik et al., (2010)). To address this, intra-event residuals (ε_{intra}) were scaled down to the single-site intra-event standard deviation (ϕ_{SS}) as shown in equations (18), (19), and (20). For ground-motion models that do not explicitly report ϕ_{SS} (e.g., Cauzzi et al., 2015; Drouet et al., 2015), an approximate value of 0.75ϕ was adopted in this study.

$$\phi = \sqrt{\phi_{S2S}^2 + \phi_{SS}^2} \quad (17)$$

$$\varepsilon_{intra} = N(0, \phi^2) \quad (18)$$

$$\varepsilon_{intra-SS} = \varepsilon_{intra} \left(\frac{\phi_{SS}}{\phi} \right) \quad (19)$$

$$\ln PGA_{RotD50} = f(M, R, \theta) + \varepsilon_{inter}\tau + \varepsilon_{intra-SS}\phi_{SS} \quad (20)$$

Where, $f(M, R, \theta)$ represents the mean prediction of the ground motion model.

3.3.2. Modeling of site effects

The site conditions assumed in the PSHA model differ significantly from those affecting the actual observation sites. The PSHA model was developed for a reference of site $V_{s30}=800$ m/s, whereas the EAU instruments, for example, are influenced by site-specific soil characteristics and soil–structure interaction (SSI) at the reactor building foundations. Consequently, a direct comparison between ground motions simulated accordingly to models characterizing the PSHA branches and those recorded by the EAU system requires adjustments to account for these site effects.

This study incorporates results from a confidential EDF report that quantified the filtering impact of reactor building–soil interaction on EAU accelerometric data relative to free-field surface ground motion. Using SSI computations performed with Code_Aster Aster (EDF, 2024) and soil modeling through Pro-MISS3D (Vandeputte, Viallet, Devesa, & Clouteau, 2020) on real earthquake recordings, the report defines an SSI filtering effect for each EDF site. This effect is expressed as a filtering coefficient K_{SSI} , characterized by its mean and standard deviation, which translates EAU-recorded motions to free-field conditions.

Additionally, an extra coefficient K_{soil} , derived from EC8 (CEN, 2004) and the V_{s30} value of each site, is applied to translate ground motion from the site-specific free-field condition to the PSHA reference. The translation of simulated motions from the PSHA reference condition (PGA_{ref}) to the EAU site condition (PGA_{site}) is performed for every simulated IM using equation (21):

$$PGA_{site} = PGA_{ref} \left(\frac{K_{SSI}}{K_{soil}} \right) \quad (21)$$

Where K_{SSI} is sampled from the distribution defined for each site. A summary table of site conditions and coefficients for all EDF observation sites is provided in the annex.

For the EPOS stations considered in this study, site effects were introduced using site terms derived from the regression analysis of Traversa et al. (2020), which characterizes ground motion across French territory. Although the reference condition of this model does not exactly match the PSHA reference condition, the regionalized regression provides a more accurate representation of attenuation and source parameters, ensuring a robust estimation of site

effects. This robustness is further supported by the large dataset used in the analysis. The site effect term z is incorporated according to equation (22):

$$PGA_{site} = \exp(\log(PGA_{ref}) + z) \quad (22)$$

A summary of the site effects as a function of Vs_{30} is included in the annex.

3.3.3. Definition and assessment of station operational time frames

Recording stations at the observation sites are not always operational, resulting in a potentially incomplete reference databases of observed ground motions. This inconsistency echoes the challenges posed by site-effects, namely that the ground motions simulated by sampling from each branch of the models do not account for the possibility of an earthquake going undetected at a site due to equipment malfunctions, maintenance, communication issues, environmental factors, and other disruptions. Consequently, the two sets of data are incompatible for comparison if compared directly.

Recognizing the issue of "holes" in recorded data, or periods where recording stations have not been operational, studies like Weatherill (2023) have suggested methods such as mixed effect regression analysis to fill these gaps. This study, however, introduces an alternative approach to reconcile the discrepancies between recorded data and PSHA-generated simulations that are not subject to operational breaks.

To estimate correlated exceedance counts, it is necessary to derive their probability structures for each network configuration active during the evaluation period, reflecting the subsets of observation sites operational at any given time. Because the methodology (and the PSHA model) assumes earthquake occurrences are independent, the resulting estimates can be aggregated according to the historical evolution of the network. In this study, this evolution was reconstructed from station-level operational data at monthly resolution. For EDF sites, continuous operation was assumed starting from each plant's commissioning date. For EPOS-France observation sites, operational documentation was retrieved from the EPOS-France network database (<https://seismology.epos-france.fr/fr/constructeur-de-requetes-station/#/>), for the period of interest.

The operational mode of EPOS-France sites has varied over time, with a "triggered" recording mode observed until approximately 2010–2014, followed by continuous recording thereafter. **Figure 3** illustrates this evolution for station OGAG, where light gray dots represent the monthly percentage of recorded time, and the dashed line shows the yearly percentage. For this station, the transition from triggered to continuous recording is evident around 2010, as indicated by the increase in yearly average recorded time.

Due to the lack of detailed information for the triggered period, a criterion was established to identify months when stations were considered operational. A station was deemed operational if it registered more than four records in each month or if the monthly recording percentage exceeded 50%. Months with fewer than four records or less than 50% recorded time were classified as offline. **Figure 4** illustrates the monthly evolution of the number of operational stations in the network of observation sites considered in this study, following the criteria described above. A sensitivity analysis on these criteria is presented in the discussion.

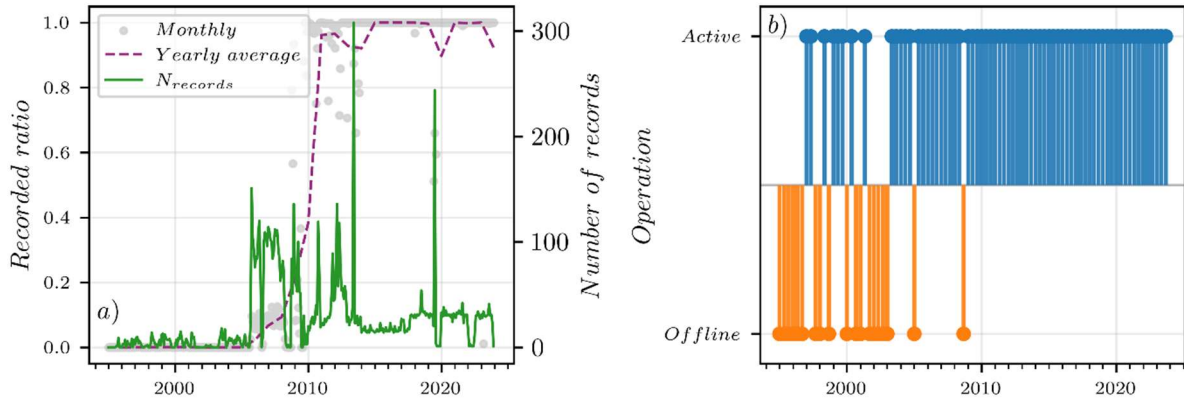


Figure 3. Characterization of the operational periods for station OGAG of the EPOS-France network. (a) – Evolution of the monthly percentage of recorded time (left axis) and number of segments recorded (right axis). Gray dots indicate the monthly percentage of recorded time, while the dashed line shows the yearly moving average, highlighting a clear transition in recording mode - from triggered to continuous - around 2010. The green line represents the number of segments (of variable duration) per month since the station's installation. (b) – Monthly pass/fail assessment to determine whether the observation site was operational during each month following its installation.

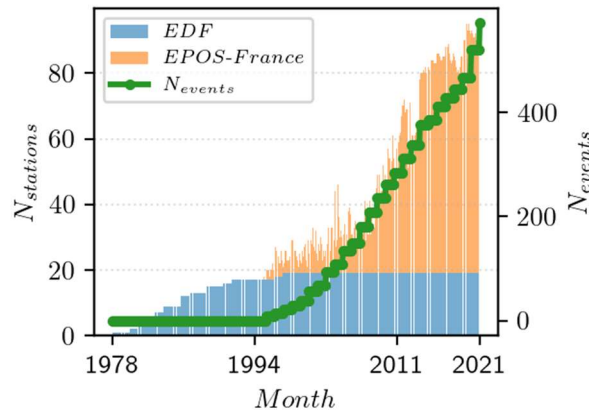


Figure 4. Evolution of the monthly availability of observation sites considered in this study. The secondary axis (N_{events}) indicates the number of events recorded by the network as a function of time.

3.3.4. Construction of the correlated distribution of exceedances for evaluation of logic tree branches

Once the simulated ground motions have been adjusted for site conditions and the network's operational history has been modeled, the correlated exceedance distributions predicted by the evaluated model can be estimated. This estimation was performed for each unique network configuration—defined by the set of operational observation sites at any given time—and for each logic-tree branch. The procedure is summarized as follows:

- **Simulation of synthetic ground motion fields:** The OQ engine is employed to generate synthetic ground-motion fields consistent with the source and ground-

motion models characterizing the selected branch. Simulations are performed for a reference site (no site effects considered) over a predefined time span T and for enough realizations to ensure a stable estimation of the distribution parameters.

- **Correction and filtering of the synthetic ground motion fields:** The simulated ground motion fields are corrected so that only the single-station intra-event variability is considered.
- **Adding site effects:** Apply site-specific adjustments detailed in section 3.3.2.
- **Modelling network evolution:** Identify the monthly sets of observation sites based on available information regarding network operability, as detailed in section 3.3.3. **Erreur ! Source du renvoi introuvable.**
- **Counting exceedances:** For each simulated realization or catalog, count the number of exceedances of the IML of interest.
- **Fitting the Pólya distribution:** Fit the Pólya distribution to the exceedance counts extracted from each realization. In cases where no exceedances were simulated across all realizations—or where the number of simulated exceedances was insufficient to ensure a stable fit, such as during early months or for very high IMLs—the exceedance distribution was treated deterministically, adopting a value of zero for the mean exceedance rate. This fallback was not required for the intensity measure levels analyzed in this study, for which the simulated catalogs provided sufficient exceedances to obtain stable parameter estimates.
- **Scaling to the evaluation period:** Scale the fitted Pólya distribution from the simulation span T to the actual evaluation period defined by the observed reference ground-motion database, following equation (16).

The synthetic ground-motion fields used in this study were simulated over a 1,000-year period and comprise 3,000 realizations. **Figure 5(a)** shows the goodness-of-fit of the correlated exceedance counts to the Pólya distribution for July 2020, assessed through the QQ plot. These counts were previously scaled from the 1,000-year simulation period to the monthly scale adopted in this study, as described in the preceding methodology section. As shown, the Pólya distribution provides an adequate representation of the empirical exceedance counts for the considered network.

Figure 5 (b) examines the stability of the estimated correlation factor as a function of the number of realizations. A secondary axis (on the right) indicates the percentage of realizations for which independence—represented by a Poisson distribution—was observed in the synthetic ground-motion fields. The figure highlights that, for the considered simulation configuration and month, the fitted correlation coefficient remains stable, and no case exhibited statistical independence. A similar plot for an early month, where independence was observed, is provided in the annex.

These analyses confirm both the suitability of the Pólya distribution for modeling the simulated data and the robustness of the correlation estimates. Stability of the adjusted distributions was verified for all months, though only selected results are shown here for brevity.

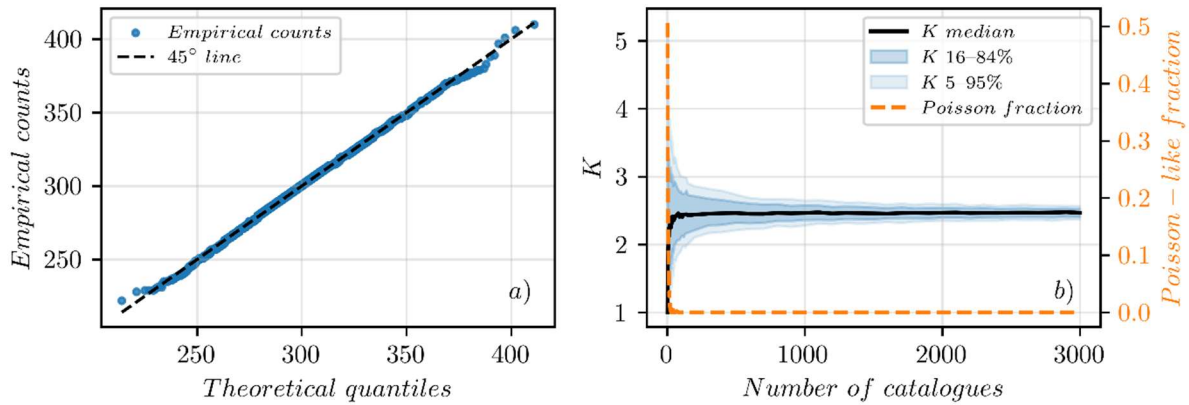


Figure 5. (a) - Goodness-of-fit of simulated correlated exceedance counts to the Pólya distribution for simulations conducted according to branch #7 of the EDF source model combined with the A17 ground-motion model using a QQ plot, and for the set of stations available in July 2019. (b) - Stability of the fitted distribution as a function of the number of realizations (catalogs). Shaded areas represent different percentiles of the estimated correlation coefficient obtained through bootstrapping. A secondary axis shows the fraction of simulated catalogs for which independence—represented by a Poisson distribution—was observed (dashed line).

Figure 6 summarizes the evolution of the distribution of correlated exceedances for an IML of 0.01g and for the logic-tree branches corresponding to the EDF source model and the ASK14 ground-motion model (a total of 100 branches). **Figure 6(a)** and (b) illustrate the evolution of the correlation parameter and the mean of the fitted distribution, respectively, while **Figure 6(c)** and (d) present the cumulative effective observation time and the cumulative number of exceedances over the evaluation period. Shaded areas represent percentile ranges derived from the population of branches considered. Reported mean values were computed without applying the logic-tree branch weights to isolate the effect of the methodological steps.

Figure 6(c) and (d) also include results for two alternative station subsets obtained through the same methodology: one considering only EDF observation sites and another including only sites that satisfy the independence assumption (i.e., subsets of monthly operational networks that maximize effective observation time and yield $K \approx 1$). These comparisons aim to assess the contribution of different operators and to quantify the gain in statistical significance achieved by explicitly modeling spatial correlation.

The results indicate that EDF's observation sites account for nearly all the statistical significance of the full network, as the difference in effective observation time compared to the entire set of observation sites is negligible. This is due to the wide spatial distribution of EDF sites, which leads to low correlation coefficients and, consequently, a higher degree of independent information from each site (see Equation(16)). In contrast, EPOS-France observation sites are primarily clustered in areas of higher seismicity within the territory, which increases correlation and reduces their incremental contribution to statistical significance.

The comparison with the independent set of observation sites highlights the benefit of explicitly modeling spatial correlation, effectively doubling the statistical significance of the analyzed network. This improvement translates into a longer equivalent evaluation time, enabling the assessment of the PSHA model for more significant return periods relevant to engineering applications.

Regarding the comparison of exceedance counts, the cumulative counts presented in **Figure 6(d)** were obtained through Monte Carlo sampling for each branch. Specifically, for a given month, the distributions corresponding to all prior months were sampled independently and aggregated to produce a single realization for the month of interest. By repeating this procedure multiple times, the distribution of cumulative counts for each branch is constructed following the historical evolution of the network of observation sites. A representation of these cumulative distributions, and their fit to the Pólya distribution, is shown in the annex of this document for a series of randomly selected branches. The results shown in **Figure 6(d)** indicate that the family of branches shown in the figure consistently overestimates the observed counts throughout the evaluation period. The observed counts represent all stations considered in this study, while results for the EDF and independent subsets are provided for comparison. This discrepancy is likely influenced by assumptions regarding the operational status of EPOS stations; this aspect will be examined in detail in subsequent sections.

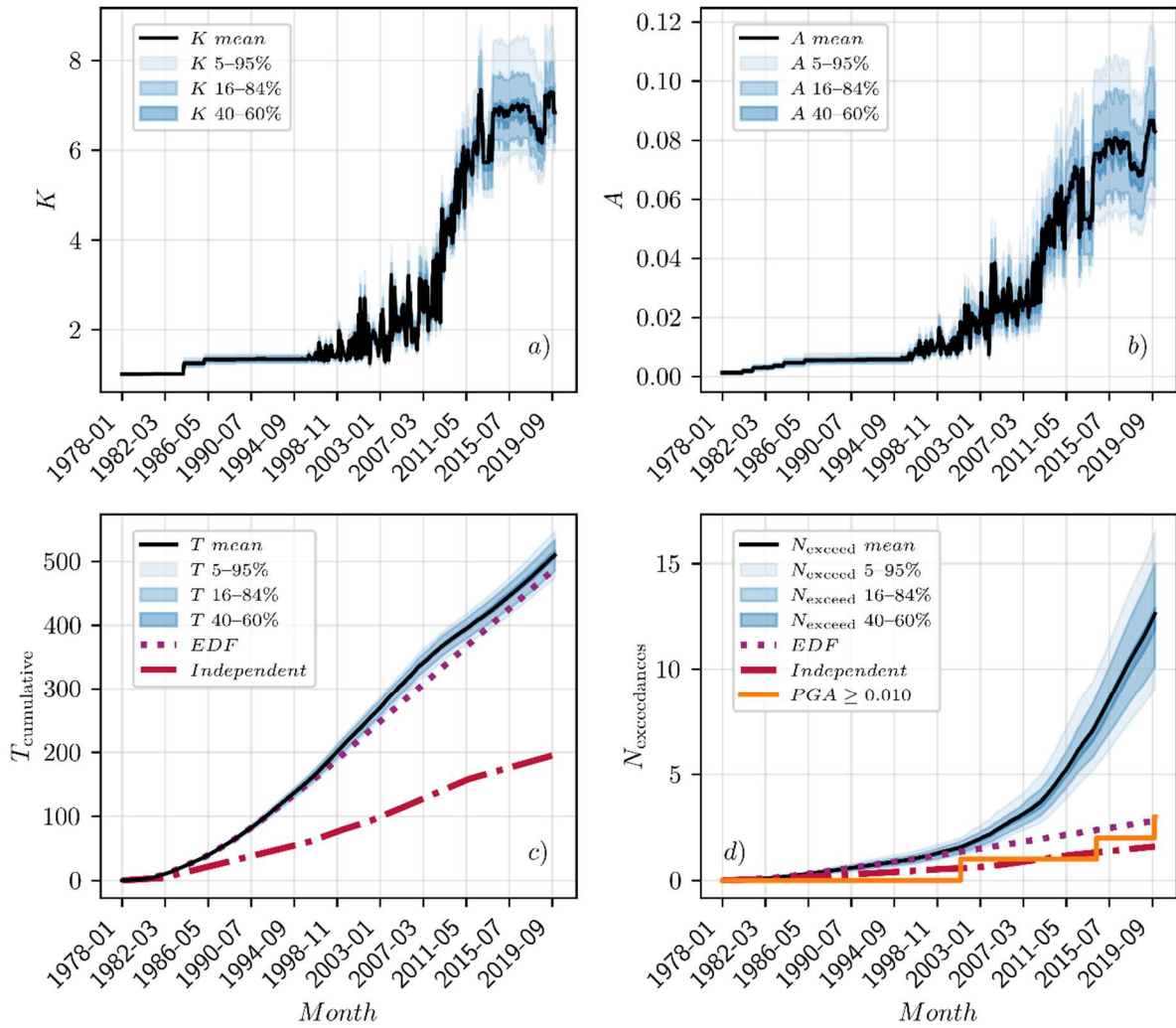


Figure 6. Evolution of the distribution of correlated exceedances for an IML of 0.01g across 100 logic-tree branches corresponding to the EDF source model and the A17 ground-motion model. (a) Correlation coefficient; (b) Mean of the distribution of correlated counts; (c) Cumulative effective observation time for the family of branches (solid line), EDF observation sites (dotted line), and a subset of sites selected to satisfy the independence assumption while maximizing cumulative observation time (dashed line); (d) Cumulative number of exceedances for the same sets, with the observed exceedance count from the reference ground-motion database (complete network) shown as an orange step line. Shaded areas represent percentile ranges across the branches considered. Reported means were computed without applying logic-tree weights.

3.4. Evaluation for a single IML

The evaluation of the PSHA model for a single Intensity Measure Level (IML) is carried out by repeating the procedure described in the previous section for all branch families. **Figure 7** presents a summary of these results for the complete set of observation sites at an IML of 0.01 g.

The results indicate that most branches tend to overestimate the observed number of exceedances for this threshold ($N_{obs} = 3$), with cumulative values ranging from 0 to a maximum of approximately 50 exceedances over a period of 42 years. When computing the likelihood of each branch reproducing the observed exceedance count, the majority fall within the lowest likelihood range, relative to the maximum possible likelihood (corresponding to a distribution whose mean matches the observed value and with the minimum correlation, $K=1$; see Figure 7 (c)).

Applying equation (1) to update branch weights based on these observations yields the distribution shown in Figure 7(b). In this figure, original weights assigned to each branch are represented by gray markers, while updated weights are shown in colors corresponding to the ground-motion prediction model associated with each branch. As expected, branches producing exceedance counts closest to the observed values receive higher weights, whereas those with larger discrepancies are penalized.

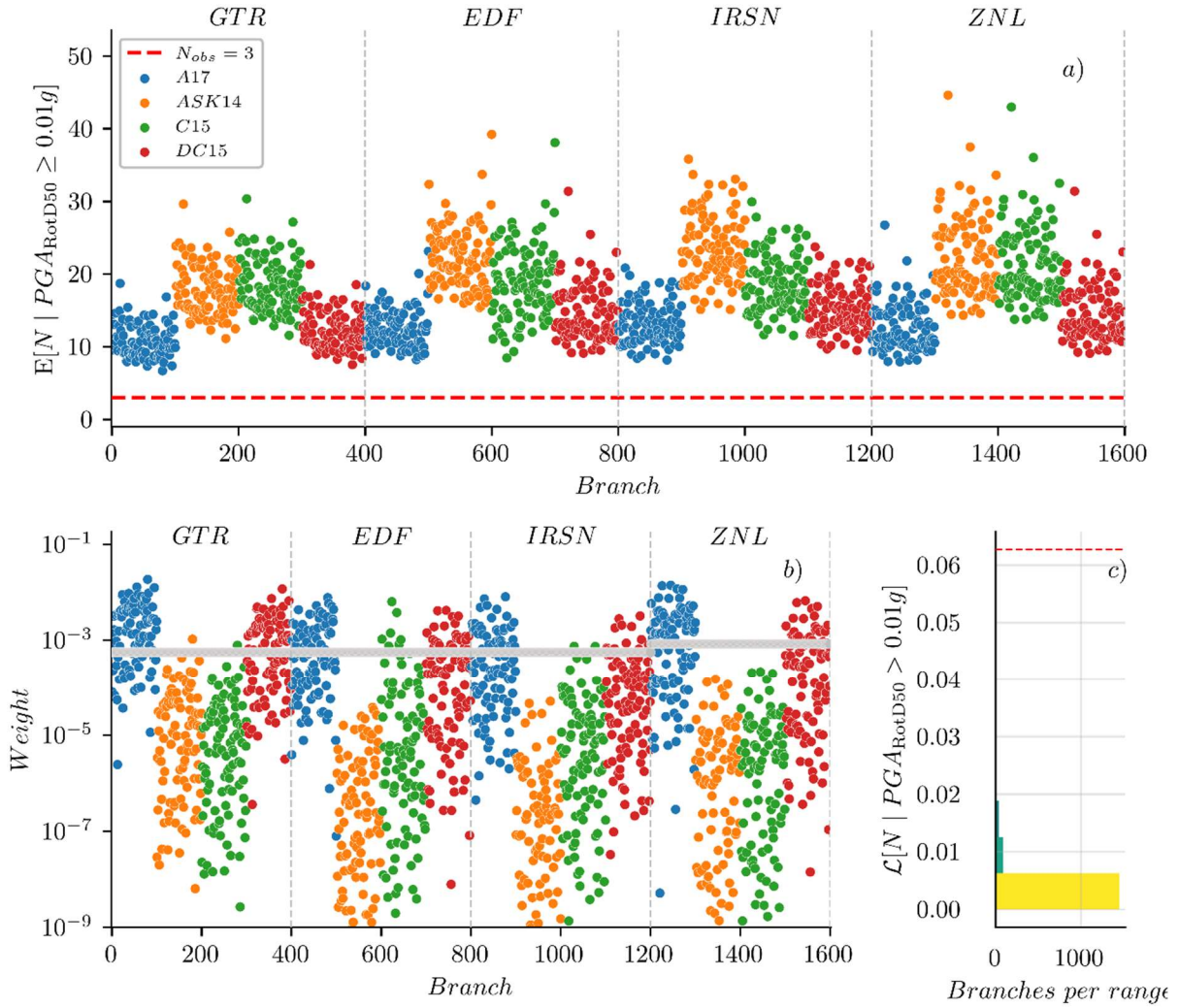


Figure 7. (a) – Comparison of expected number of exceedances for each branch in the logic tree (markers) and the observed number of exceedances in the reference database (dotted line), (b) – Distribution of weights for each branch in the logic tree. The gray dots represent the originally assigned (prior) weight of each branch and the markers represent the updated (posterior) weight of each branch. (c) – Histogram of the likelihood of the logic-tree branches with respect to the observed number of exceedances in the reference database, the red dotted line, here included for reference purposes, represents the maximum possible likelihood, computed from the Pólya distribution defined by a mean value matching the observed number of exceedances, and the lowest correlation factor computed from the branches of the logic tree model. These results were obtained from the evaluation of the PSHA model for the complete set of observation sites, for a PGA_{RotD50} IML of 0.01g.

Figure 8 illustrates the relative weights assigned to each branch before (prior) and after (posterior) the Bayesian updating process. Branches are grouped by source model, and stacked bars display the weights associated with each ground-motion model. Bars with a hashed pattern represent the original weights, while solid colors indicate the updated values. The results show that, among the source models, the GTR area-source model retains over 40% of the total weight, closely followed by the ZNL zoneless-source model. The zoneless

model and the IRSN source model contribute secondary shares, each representing approximately 10% of the total weight.

Regarding the ground-motion models, A17 consistently provides the best performance across the source models, followed by DC15. This behavior is expected given that both A17 and DC15 were developed using datasets that include a substantial amount of French and regional European recordings—particularly from areas close to the observation sites considered in this study. In contrast, ASK14 is primarily derived from the NGA-West2 global strong-motion database, which is dominated by recordings from high-seismicity regions such as California. The closer regional relevance and the inclusion of low-to-moderate-magnitude French events in A17 and DC15 therefore explain their superior performance relative to ASK14.

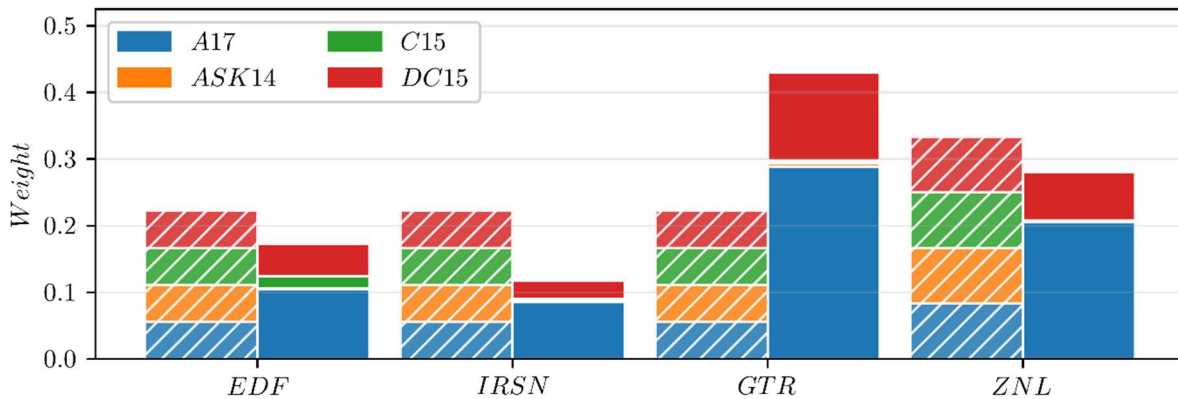


Figure 8. Comparison of the weights assigned to each branch in the logic tree for an IML of 0.01g. The weights are group by source model family. Stacked colored bars represent the cumulative weight of the branches as a function of the ground motion model. In this figure the bars with a hashed pattern indicate original (prior) weights, while solid colors represent updated (posterior) weights computed with respect to the complete set of observation sites.

3.5. Generalized evaluation for multi IMLs

The methodology for evaluating PSHA models described thus far is anchored to a specific IML, whereas in practice, the same branch weights are typically applied across a range of IMLs to construct hazard curves. This raises the question of how to integrate the inferences obtained for individual IMLs into a single, consistent estimation of updated weights for the logic tree model.

In this study, the likelihood function associated with each branch of the logic tree is formulated following the principles of Bayesian updating. Specifically, when multiple IMLs are considered simultaneously, the likelihood is represented using a multivariate probability distribution. The use of a multivariate framework is necessary because exceedances at different discrete IMLs are not statistically independent; rather, they exhibit a clear degree of correlation, as illustrated in **Figure 9**, which shows the relationship between exceedance counts across all branches for different IMLs.

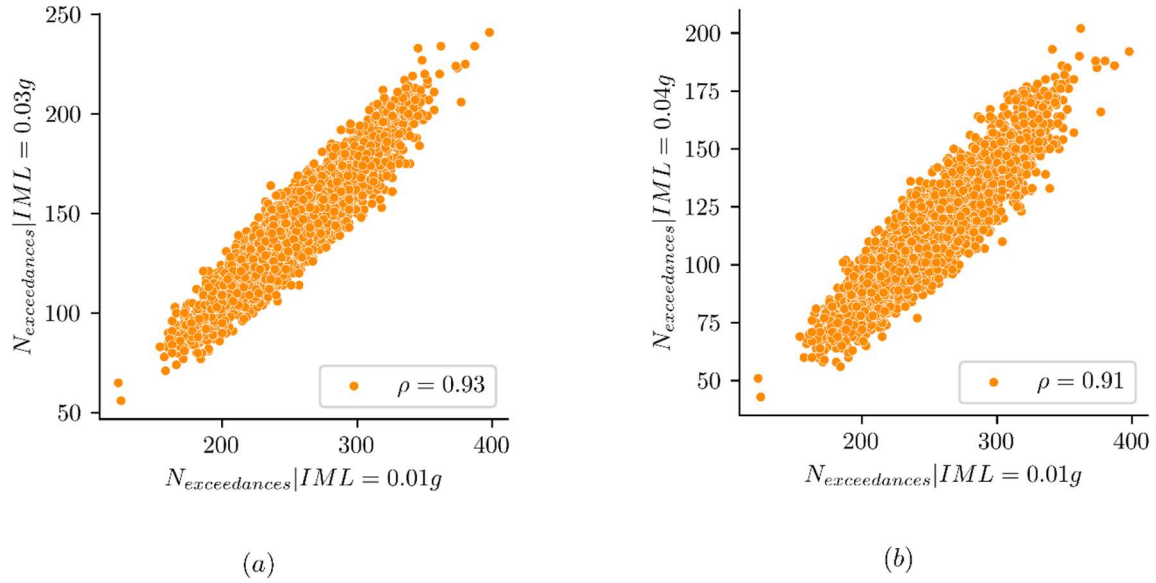


Figure 9. Evaluation of the correlation between the number of exceedances obtained for (a) – IMLs: 0.01g and 0.03g, and (b) - IMLs: 0.01g and 0.04g. The results correspond to one of the logic tree branches (EDF source model – DC15 GMM), for the complete set of observation sites, and for the original simulation period of 1000 years.

To capture the dependence structure among IMLs, the multivariate distribution is constructed by combining the marginal probability distributions defined individually for each IML with a Bernstein copula (Bernstein, 1912). The copula serves as a flexible tool for modeling joint dependence while preserving marginal characteristics. The Bernstein copula is adopted in this study because of its nonparametric, data-adaptive nature, which accurately reproduces complex and nonlinear correlations among IML exceedances without imposing restrictive assumptions on dependence form. Unlike traditional parametric copulas (e.g., Gaussian or Clayton), the Bernstein copula uses polynomial approximations that provide a smooth and continuous representation of the empirical dependence observed in simulated data. This flexibility is particularly advantageous for seismic applications, where correlations between ground-motion intensity measures vary with magnitude, distance, and site conditions. Moreover, the Bernstein copula ensures uniform marginals and numerical stability, both essential for consistent likelihood evaluation within the Bayesian updating framework. By doing so, the constructed likelihood function faithfully reproduces the correlation patterns observed in the simulated ground-motion data.

Because the observed data consist of discrete counts, evaluating a continuous point density at integer values is not meaningful. Instead, the probability of observing a specific count vector is computed as the copula-induced probability mass of the corresponding hyper-rectangle in the unit hypercube. This approach uses inclusion–exclusion over all corners of the hyper-rectangle to obtain the exact joint probability. For two dimensions, this simplifies to the well-known four-corner formula. This “rectangle probability” method is the correct adaptation of copula modeling for discrete marginals: it preserves the marginal Pólya distributions and computes the exact joint mass at integer observations, avoiding near-zero artifacts that occur when evaluating a continuous density at lattice points. The resulting joint likelihood is then combined with prior branch weights to produce posterior weights across source families and

GMMs, enabling coherent multi-IML updating while respecting the discrete nature of exceedance counts.

Figure 10 illustrates the number of exceedances across all observation sites for each branch and for combined branches under different weighting schemes for a range of IMLs ranging from 0.01g to 0.05g: the original weights (N_{wi}), the updated weights for each individual IML ($N_{wf(IML)}$), and the updated weights obtained by computing the joint likelihood across all IMLs considered in this study ($N_{wf(joint)}$). The improvement in predicted exceedances compared to estimates using the initial weights demonstrates the overall efficiency of the Bayesian updating technique.

In the single-IML Bayesian update, posterior weights are computed by maximizing the likelihood of the observed exceedance count at a specific IML. This likelihood is based on the full Pólya distribution for each branch, rather than forcing the expected count to match the observation exactly. Consequently, the optimization is local: it improves fit at one threshold without ensuring consistency across other thresholds. This can lead to overfitting or under-adjustment, particularly when observed counts are small or branch distributions exhibit high dispersion.

In contrast, the joint update maximizes the likelihood of the entire vector of observed exceedances across all IMLs simultaneously. This global optimization favors branches that reproduce the overall decay pattern of exceedances, reducing bias and improving coherence across thresholds. When observations follow a smooth trend, the joint update often achieves better alignment with the observed pattern—even at individual IMLs—than single-IML updates, which optimize locally but ignore global consistency.

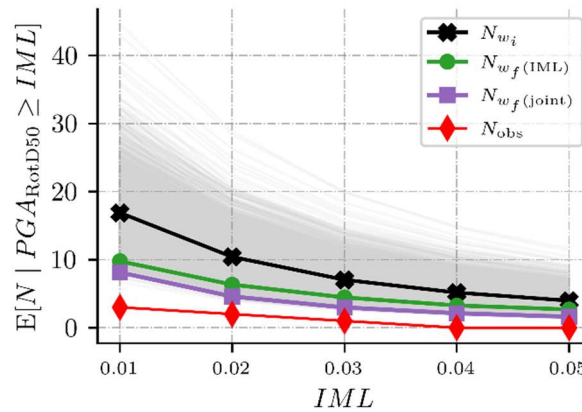


Figure 10. Estimated number of exceedances for different IMLs across all observation sites. Gray lines represent the estimates from each branch of the logic tree. The black line with “x” markers shows the mean prediction using the original (prior) weights. The green line with circular markers corresponds to the combination using updated (posterior) weights obtained separately for each IML, while the violet line with square markers represents the combination using updated weights derived from the joint probability distribution across all IMLs. The observed number of exceedances is shown in red with diamond markers.

4. Discussion

The methodology adopted in this study relies on the simulation of ground-motion fields to numerically estimate the distributions of correlated exceedances for one or more IMLs, thereby enabling the evaluation and adjusting of the weights assigned to each branch of the logic tree through Bayesian updating. Results presented in previous sections confirm that the Pólya distribution provides a suitable framework for characterizing this variability. However, because PSHA models are typically developed for a reference site condition, it is essential to incorporate site-specific effects accurately. Furthermore, acknowledging the imperfect operational history of observation sites is critical to ensure that the simulated synthetic fields yield estimates that are comparable to empirical observations in reflecting this history.

Among these two primary sources of uncertainty, the characterization of operational periods emerges as the most influential. This is particularly relevant in the present case, where several observation sites belonging to the EPOS-France network operated under a triggered recording mode during part of the evaluation period. In this mode, instruments only record data when ground motion exceeds a predefined threshold. Unfortunately, detailed information regarding both the triggering thresholds and the operational timelines of these stations is not readily available. Consequently, as outlined in Section 3, a series of informed assumptions were introduced to approximate station operability. These assumptions include considering that the IMLs analyzed in this study exceed the triggering thresholds. To assess the sensitivity of the results presented earlier, the logic-tree branches evaluated in Figure 6 were re-analyzed under an alternative set of assumptions regarding station operation. The results labeled EPOSB were obtained by applying a stricter validation criterion: a minimum of 10 records per month was required to confirm the operation of an observation site (compared to the threshold of 4 records used in previous sections). The override rule—allowing validation when the number of records falls below the minimum—was retained, provided that at least 50% of the month was recorded.

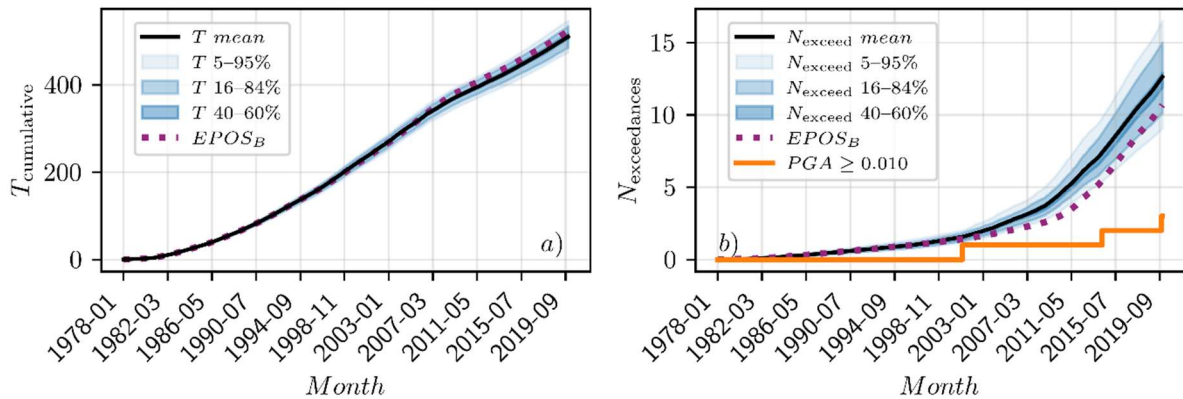


Figure 11. Evolution of correlated exceedance distributions for an intensity measure level (IML) of 0.01 g across 100 logic-tree branches, based on the EDF source model combined with the A17 ground-motion model, under two alternative assumptions regarding EPOS-France station operational status. (a) Cumulative effective observation time for branches assuming (i) a minimum of four records per month or at least 50% monthly coverage (solid line), and (ii) a minimum of two records per month or at least 50% monthly coverage (dashed line). (b) Cumulative number of exceedances for the same configurations, with the observed

exceedance count from the reference ground-motion database (complete network) shown as an orange step line. Shaded areas indicate percentile ranges across branches. Reported mean values were computed without applying logic-tree weights.

The results presented in Figure xx indicate that even under substantially more rigorous assumptions regarding station operability—such as increasing the minimum monthly record requirement from four to ten—the estimated cumulative number of exceedances remains significantly lower than the observed count for the considered IML. This outcome suggests that uncertainties related to operational periods, while influential, cannot fully account for the discrepancy between simulated and observed exceedances for the complete set of observation sites.

Alternatively, the promising evaluation periods obtained when considering only EDF observation sites suggest that restricting the analysis to this subset may provide more reliable results without compromising the statistical significance of the inferences. Indeed, as shown in **Figure 6**, the spatial distribution of EDF sites ensures that their contribution to effective observation time is nearly equivalent to that of the complete network. **Figure 12** illustrates the results obtained when evaluating the PSHA model using only EDF sites. These results indicate that the predicted cumulative number of exceedances aligns more closely with the observed values. A figure with the complete evaluation for all family branches and considering EDF observation sites is included in the annex.

Complementarily, **Figure 13** compares prior and posterior weights updated using different sets of observation sites for the joint range of IMLs considered in this study (0.01 g–0.05 g). The results indicate that incorporating the complete set of sites leads to a markedly unbalanced distribution of weights, where, for example, the EDF area source model—and particularly the ASK14 ground-motion model—consistently absorb most of the weight. In contrast, using only EDF sites yields a more balanced distribution, more consistent with the original weight set. This finding, combined with the considerable uncertainty regarding the operational history of EPOS-France stations, raises an important question about whether these sites should be included in the evaluation of the PSHA model.

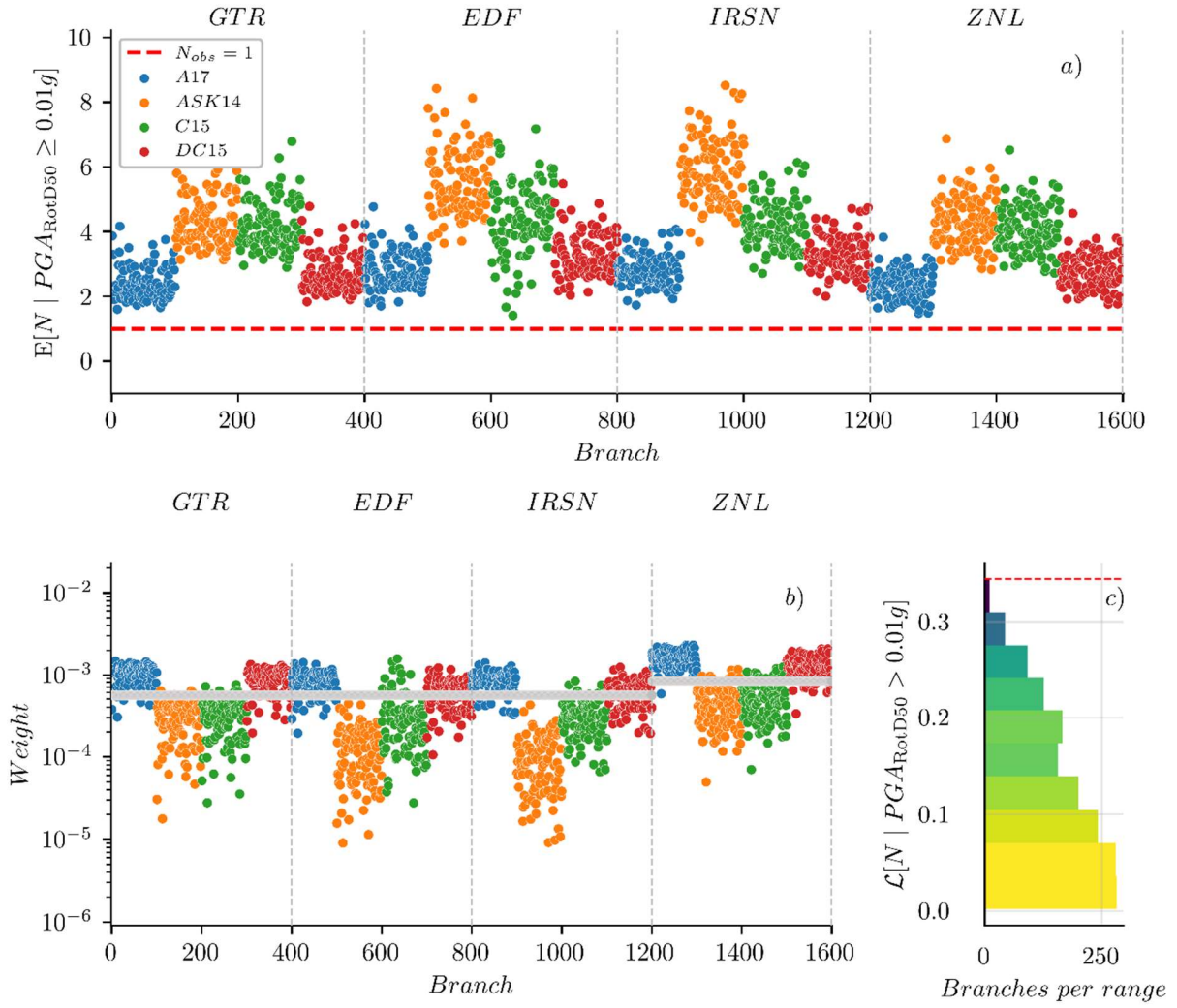


Figure 12. Evolution of the distribution of correlated exceedances for an intensity measure level (IML) of 0.01 g across 100 logic-tree branches corresponding to the EDF source model combined with the A17 ground-motion model, considering only stations located at EDF operational sites. (a) Estimated correlation coefficient; (b) Mean of the fitted distribution of correlated exceedance counts; (c) Cumulative effective observation time for the family of branches (solid line), EDF observation sites (dotted line), and a subset of sites selected to satisfy the independence assumption while maximizing cumulative observation time (dashed line); (d) Cumulative number of exceedances for the same configurations, with the observed exceedance count from the reference ground-motion database (EDF observation sites only) shown as an orange step line. Shaded areas represent percentile ranges across the branches considered. Reported mean values were computed without applying logic-tree weights.

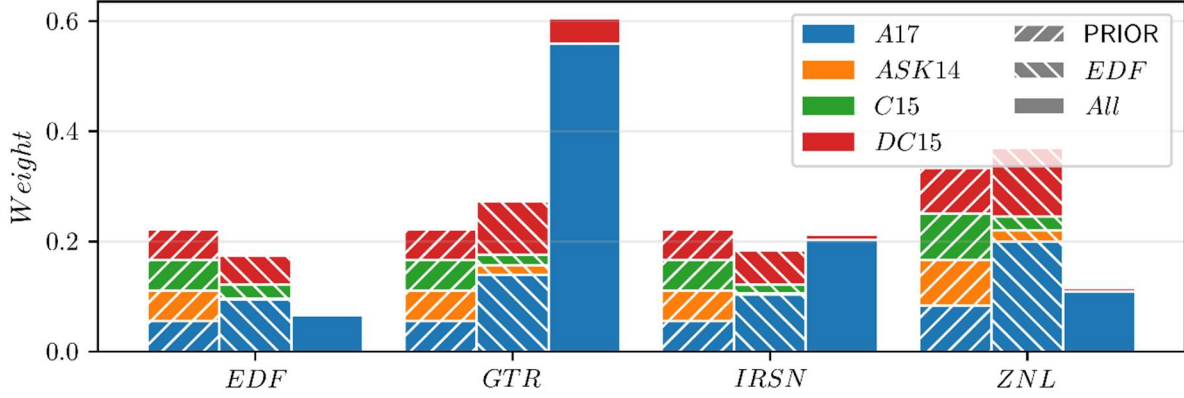


Figure 13. Comparison of weights assigned to each branch in the logic tree for the joint IML range (0.01 g–0.05 g), grouped by source model family. Stacked colored bars represent the cumulative weight of branches as a function of the ground-motion model. Bars on the left with a hashed pattern indicate the original (prior) weights. Bars in the center, with an opposite hash pattern, represent the updated (posterior) weights computed using EDF observation sites, while solid-colored bars on the right represent updated (posterior) weights based on the complete set of observation sites.

If sources of uncertainty considering the operation of the observation sites were adequately addressed, an alternative approach could be adopted to maximize the statistical significance of the inferences drawn from this methodology. Specifically, by characterizing the correlation coefficient through a proxy for spatial correlation, it would be possible to optimize the selection of stations that maximize effective observation time across all branches. This optimization could be achieved iteratively, as an exhaustive evaluation of all possible station combinations for each month is computationally infeasible. **Figure 14** illustrates such a proxy—the mean interstation distance (see Equation(23))—computed for several source and ground-motion models and for selected IMLs. As shown, this proxy provides a reliable predictor of the correlation parameter, thereby simplifying the optimization problem. Following the rationale of the proposed methodology, the optimal set of stations could be determined by maximizing the effective observation time across all branches using an appropriate statistic (e.g., mean or percentile). It is important to emphasize that the selected set of stations must remain consistent across all branches to ensure a coherent basis for comparison. Optimizing the configuration for each branch individually would yield branch-specific detectability rather than a uniform evaluation framework.

$$\bar{d} = \frac{2}{S(S-1)} \sum_{1 \leq i < j \leq S} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \quad (23)$$

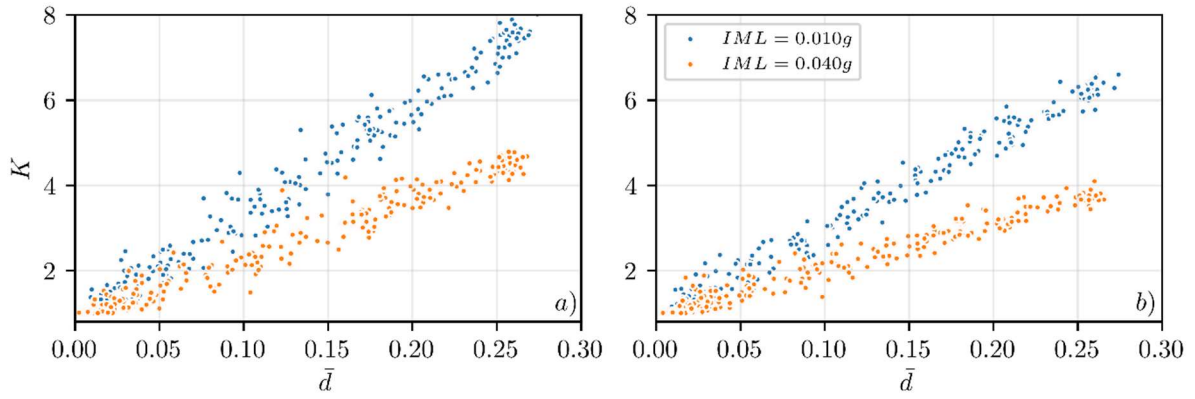


Figure 14. Spatial correlation factor as a function of the mean pairwise distance between stations in the complete set of observation sites for selected IMLs considered in this study. (a) EDF area-source model; (b) ZNL (zoneless) source model.

Where, x_i and y_i represent the coordinates of each observation site in the plane, and SSS denotes the total number of sites considered.

5. Conclusion and perspectives

This study proposes and demonstrates a comprehensive framework for the direct evaluation and Bayesian updating of PSHA logic-tree weights that explicitly accounts for the global spatial correlation of ground motions across multiple observation sites. The methodology integrates event-based simulations to generate branch-consistent synthetic ground-motion fields, harmonization of these to station conditions through site-effect corrections, explicit modeling of station operability over time, and likelihood-based updating using Pólya (negative binomial) marginals for correlated exceedance counts. For multi-IML updating, a Bernstein copula is employed to capture the dependence structure among discrete exceedance counts while preserving the marginal distributions. Applied to the metropolitan France PSHA model (Drouet et al., 2020; Weatherill et al., 2022), the proposed framework demonstrates that the Pólya captures the variance inflation caused by spatial correlation. Goodness-of-fit diagnostics, including QQ plots and bootstrap analyses, consistently reveal correlation effects ($K > 1$) at monthly resolution, with independence (Poisson behavior) rarely observed, primarily during early periods of very limited network coverage. The incorporation of the correlation factor K and the associated notions of effective station number and effective observation time, S_{ess} and T_{ess} , offers a physically interpretable measure of informational gain under dependence.

It should be emphasized that the simulated ground-motion fields used in this study are not non-ergodic simulations; at best, they are site-specific approximations (with site effects imported from an independent model and single-station intra-event variability compiled across GMMs). In contrast, the observations reflect the non-ergodic, space- and time-variable reality recorded by the network during the evaluation period, so all model–data comparisons should be interpreted in light of this mismatch.

Empirically, the EDF observation sites—owing to their wide spatial deployment—contribute nearly the same effective observation time as the full network, despite the larger nominal size of the latter. In contrast, the EPOS-France sites tend to be clustered in higher-seismicity regions, which elevates correlation and reduces incremental statistical power. At the IML of 0.01 g, most branches overpredict the cumulative exceedance counts relative to observations; the Bayesian update penalizes such branches, with combinations involving A17 and the GTR/ZNL source models typically outperforming others, while ASK14 is least representative. This ranking is consistent with data provenance: A17 is calibrated on the pan-European RESORCE database with a substantial French component and explicit treatment of French/Swiss small-magnitude behavior, whereas ASK14 is derived from the NGA-West2 global strong-motion database (heavily informed by active-crustal regions such as California), making it less tailored to the low-to-moderate-seismicity French context considered here. Single-IML updates improve local fit but can be inconsistent across thresholds, whereas joint multi-IML updating better reproduces the observed decay pattern of exceedances, yielding more coherent posterior weights. Sensitivity analyses on station operability indicate that even substantially stricter assumptions (e.g., requiring ≥ 10 monthly records with a 50% of monthly available recordings override) do not reconcile the full-network discrepancy between simulated and observed counts, pointing to the limits of operability heuristics. Moreover, when the full network is used for the joint IML case, posterior weights can become markedly unbalanced (e.g., large absorption by A17 in specific families), while restricting evaluation to EDF sites produces more balanced weights that are closer to the original priors and better aligned with engineering expectations. Taken together, explicitly modeling spatial dependence and performing joint multi-IML updating increases effective statistical significance—on the order of a twofold gain—enabling evaluation at longer equivalent return periods of practical relevance.

5.1. Limitations

The implementation necessarily relies on several assumptions and data constraints. Operability of EPOS-France stations in the triggered-recording era is inferred via monthly heuristics because trigger thresholds and downtime logs are incomplete; these choices influence the estimated correlation K and the effective time T_{ess} . Site-effect corrections combine K_{SSJ} and K_{soil} factors for EDF sites and site-to-site repeated effects from GMM regression for sites belonging to the EPOS-France network; some GMMs lack explicit ϕ_{SS} , for which a 0.75ϕ approximation is adopted, and residual partitioning between ergodic and non-ergodic components is simplified. Catalog construction choices—declustering parameters, the minimum magnitude threshold ($M_w \geq 4.5$), and the evaluation window (1977–2020)—all influence the behavior of the evaluation, particularly at low IMLs. The declustering and magnitude threshold were selected to ensure consistency with the PSHA model under study, whereas the evaluation window reflects a constraint imposed by the availability of observed reference data rather than a model criterion. Earthquakes are modeled as a Poisson process (mainshocks only), with uncorrelated inter-event residuals; aftershocks/foreshocks and potential non-stationarities are excluded. Finally, the Bernstein copula is selected for flexibility with discrete marginals via rectangle probabilities; alternative dependence structures could also be considered. These limitations delineate the scope of inference and indicate priorities for data curation and methodological refinement.

5.2. Perspectives

Several avenues can consolidate and extend the present framework. First, improved data stewardship for station operability is essential: integrating station-level trigger thresholds, downtime/maintenance logs, and mode transitions would allow replacing heuristics with an explicit detectability model that couples simulated fields to realistic recording thresholds and noise floors. Second, advances in spatial correlation and site effects should be pursued, including non-ergodic or regionalized GMMs, refined estimates of ϕ_{SS} for models lacking explicit values, and hierarchical Bayesian treatments of K that vary across branches and months with priors informed by network geometry. Third, multi-IML updating would benefit from systematic uncertainty quantification—posterior predictive checks, credible intervals on weights and exceedance predictions—and comparative assessments of copulas (e.g., vine constructions) for discrete marginals. Fourth, the interstation-distance proxy introduced in the previous section can guide optimization of station subsets to maximize T_{ess} under a branch-agnostic, month-wise configuration fixed across branches, ensuring a coherent evaluation baseline; computationally tractable algorithms (e.g., greedy or metaheuristic) can implement this selection. Fifth, broader validation—other IMs (e.g., SA at key periods), regions, and extended periods—will test transferability and support regulatory applications, informing guidance on the inclusion/exclusion of stations with uncertain operability. Finally, reproducibility should be enhanced by packaging the workflow (simulation, correction, counting, fitting, scaling, updating) into documented scripts and OQ-compatible modules, with transparent parameter tables and annexed sensitivity analyses.

In summary, the proposed correlation-aware, multi-IML Bayesian evaluation and updating framework offers a principled path to data-informed logic-tree weighting. By explicitly accounting for spatial dependence, harmonizing site conditions, and coherently aggregating information across thresholds, it improves the statistical robustness and practical relevance of PSHA evaluation while providing clear priorities for targeted data improvements and methodological extensions.

6. Bibliography

- Abrahamson, N. A., Silva, W. J., & Kamai, R. (2014). Summary of the ASK14 ground motion relation for active crustal regions. *Earthquake Spectra*, 30(3), 1025-1055. <https://doi.org/10.1193/070913EQS198M>.
- Agency, I. A. (2024). *Evaluation of Probabilistic Seismic Hazard Analysis (PSHA) for Nuclear Installations Based on Observational Data (IAEA_TECDOC-2067)*. Vienna: IAEA.
- Aimeri, R., Cauzzi, C., & Drouet, S. (2015). Ground motion models for PSHA: A comparison between empirical and simulation-based approaches. *Bulletin of the Seismological Society of America*, 105(1), 123-145. <https://doi.org/10.1785/0120140029>.
- Al Atik, L., Abrahamson, N. A., Bommer, J. J., Scherbaum, F., Cotton, F., & Kuehn, N. (2010). The variability of ground-motion prediction models and its components. *Seismological Research Letters*, 81(5), 794–801. <https://doi.org/10.1785/gssrl.81.5.794>.
- Albarelo, D., & D'Amico, V. (2008). Testing probabilistic seismic hazard estimates by comparison with observations: An example in Italy. *Geophysical Journal International*, 175(3), 1088-1094.
- Albarelo, D., & Peruzza, L. (2017). Accounting for spatial correlation in the empirical scoring of probabilistic seismic hazard estimates. *Bulletin of Earthquake Engineering*, 15, 2571-2585.
- Alsokhon, A. (2022). *Actualisation bayésienne de l'aléa sismique et calcul du risque*. Aix-en-Provence: Arts et Métiers.
- Ameri, G., Drouet, S., Traversa, P., Bindi, D., & Cotton, F. (2017). Toward an empirical ground motion prediction equation for France: accounting for regional differences in the source stress parameter. *Bulletin of Earthquake Engineering*, 15, 4681-4717. DOI:10.1007/s10518-017-0171-1.
- Baker, J. W., & Jayaram, N. (2008). Correlation of spectral acceleration values from NGA ground motion models. *Earthquake Spectra*, 24(1), 299-317. <https://doi.org/10.1193/1.2857544>.
- Beauval, C., Bard, P., Hainzl, S., & Gueguen, P. (2008). Can Strong-Motion Observations be Used to Constrain Probabilistic Seismic-Hazard Estimates? *Bulletin of the Seismological Society of America*, 98(2), 509-520.
- Bernstein, S. (1912). Démonstration du théorème de Weierstrass fondée sur le calcul des probabilités. *Commun Kharkov Math Soc.*, 13, 1–2.
- Buscetti, M., Traversa, P., Perron, V., Rischette, P., Hollender, F., & Shible, H. (2024). Epos-France Database of Earthquake Ground Motion for Mainland France: 1996–2021 Updated Version. *Seismological Research Letters*, 96 (2A): 1214–1226. doi: <https://doi.org/10.1785/0220240053>.
- Cauzzi, C., Faccioli, E., Vanini, M., & Bianchini, A. (2015). Updated predictive equations for broadband (0.01–10 s) horizontal response spectra and peak ground motions, based on a global dataset of digital acceleration records. *Bulletin of Earthquake Engineering*, 13, 1587-1612.
- CEN. (2004). Eurocode 8: Design of Structures for Earthquake Resistance. Part 1: General Rules, Seismic Actions and Rules for Buildings. Brussels, Belgium: Comité Européen de Normalisation.
- Cornell, A. (1968). Engineering seismic risk analysis. *Bulletin of the Seismological Society of America*, 1583-1606.
- Drouet, S., & Cotton, F. (2015). Regional stochastic GMPEs in low-seismicity areas: Scaling and aleatory variability analysis—Application to the French Alps. *Bulletin of the Seismological Society of America*, 105(4), 1883-1902.

- Drouet, S., Ameri, G., Le Dortz, K., Secanell, R., & Senfaute, G. (2020). A Probabilistic seismic hazard map for the metropolitan France. *Bulletin of Earthquake Engineering*, 18(4), 1865-1898. <https://doi.org/10.1007/s10518-020-00790-7>.
- EDF. (2024). *Documentation Code ASTER*. Récupéré sur <https://code-aster.org>
- Fujiwara, H., Morikawa, N., Ishikawa, Y., Okumura, T., Miyakoshi, J., Nojima, N., & Fukushima, Y. (2009). Statistical comparison of national probabilistic seismic hazard maps and frequency of recorded JMA seismic intensities from the K-NET strong-motion observation network in Japan during 1997–2006. *Seismological Research Letters*, 80(3), 458-464.
- Gardner, J., & Knopoff, L. (1974). Is the sequence of earthquakes in Southern California, with aftershocks removed, Poissonian? *Bulletin of the Seismological Society of America*, 64, 1363-1367.
- Gorczak, K., Burzykowski, T., & Claesen, J. (2025). A hierarchical negative-binomial model for analysis of correlated sequencing data: practical implementations. *Bioinformatics Advances*, <https://doi.org/10.1093/bioadv/vbaf126>.
- Humbert, N. (2015). A Polya distribution model for spatial correlation in seismic hazard evaluations. *Geophysical Journal International*, 202(1), 345-359. <https://doi.org/10.1093/gji/ggv129>.
- Humbert, N. (2015). PSHA updating technique with a Bayesian Framework: Innovations. *CSNI Workshop on "PSHA Results and Benefit of Bayesian Techniques for Seismic Hazard Assessment"*. Pavia, Italy: Eucentre Foundation.
- Iervolino, I., Vitale, A., & Cito, P. (2021). Empirical assessment of seismic design hazard's exceedance area. *Scientific Reports*, 11(1), 18803.
- International Atomic Energy Agency. (2024). Evaluation of Probabilistic Seismic Hazard Analysis (PSHA) for Nuclear Installations Based on Observational Data, IAEA-TECDOC-2067. IAEA, <https://doi.org/10.61092/iaea.jfkj-gjuz>.
- Jayaram, N., & Baker, J. (2009). Correlation model for spatially distributed ground-motion intensities. *Earthquake Spectra*, 38:1687-1708. DOI: 10.1002/eqe.922.
- Lanzano, G., Sgobba, S., Luzi, L., Puglia, R., Pacor, F., & Felicetta, C. (2019). The pan-European Engineering Strong Motion (ESM) flatfile: compilation criteria and data statistics. *Bulletin of Earthquake Engineering*, 17(2): 561–582. DOI: 10.1007/s10518-018-0480-z.
- Mak, S., Clements, R., & Schorlemmer, D. (2014). The statistical power of testing probabilistic seismic-hazard assessments. *Seismological Research Letters*, 85(4), 781-783.
- Ordaz, M., & Reyes, C. (1999). A methodology for direct testing of probabilistic seismic hazard estimates using recorded data. *Bulletin of the Seismological Society of America*, 89(1), 13-24.
- Organisation for Economic Co-Operation and Development (OECD) / Nuclear Energy Agency. (2015). Workshop on Testing Probabilistic Seismic Hazard Analysis Results and the Benefits of Bayesian Techniques for Seismic Hazard Assessment. Pavia: EUCENTRE/IUSS.
- Pagani, M., Monelli, D., Weatherill, G., Danciu, L., Crowley, H., Silva, V., . . . Vigano, D. (2014). OpenQuake engine: An open hazard (and risk) software for the Global Earthquake Model. *Seismological Research Letters*, 85(3), 692–702. <https://doi.org/10.1785/0220130087>.
- Pagani, M., Silva, V., Simionato, M., & Johnson, K. (2023). *OpenQuake Engine Manual - Release 3.16.1*. Pavia: GEM.
- Secanell, R., & Drouet, S. (2021). *Evaluation of SERA seismic hazard assessment results using Bayesian approaches*. Paris: FUGRO.

- Stirling, M., Petersen, M., & Gerstenberger, M. (2006). Validation and update of the probabilistic seismic hazard model for New Zealand. *Bulletin of the Seismological Society of America*, 96(6), 2010-2021. <https://doi.org/10.1785/0120050257>.
- Stoklosa, J., Blakey, R., & Hui, F. (2022). An overview of Modern Applications of Negative Binomial Modelling in Ecology and Biodiversity. *Diversity*, 14(5), 320. <https://doi.org/10.3390/d14050320>.
- Traversa, P., Maufroy, E., Hollender, F., Perron, V., Bremaud, V., Shible, H., . . . Douste-Bacqué, I. (2020). RESIF RAP and RLBP Dataset of Earthquake Ground Motion in Mainland France. *Seismological Research Letters*, 91(4), 2409-2424. <https://doi.org/10.1785/0220190367>.
- Vandeputte, D., Viallet, E., Devesa, G., & Clouteau, D. (2020). Deconvolution and soil-structure interaction with Pro-MISS3D software based on seismic recordings on Hualien array. *Bulletin of Earthquake Engineering*, 18(4), 1234-1256. <https://doi.org/10.1007/s10518-020-00789-3>.
- Weatherill, G. (2022). Data and Code for the Implementation of the Drouet et al., (2020) PSHA Model for Metropolitan France into OpenQuake [Data set]. Zenodo. <https://doi.org/10.5281/zenodo.13991952>.
- Weatherill, G. (2023). Challenges in testing PSHA models with limited ground motion data. *Seismological Research Letters*, 94(3), 482-491. <https://doi.org/10.1785/0220220065>.
- Weatherill, G., Cotton, F., Daniel, G., & Zentner, I. (2022). *Implementation of the Drouet et al. (2020) PSHA for France into OpenQuake: Comparisons and Modelling Issues (SIGMA-2 Deliverable SIGMA2-2021-D5-085)*. SIGMA-2. www.sigma-programs.com.
- Weatherill, G., Cotton, F., G, D., & Zentner, I. (2023). Developments and Tools for PSHA to Data Comparison. *Bulletin of Earthquake Engineering*, 21(1), 123-145. <https://doi.org/10.1007/s10518-023-01567-8>.
- Wolfram Research, I. (2023, July 19). Récupéré sur Wolfram World: <https://mathworld.wolfram.com/NegativeBinomialDistribution.html>
- Zanetti, L., Chiffi, D., & Petrini, L. (2023). Philosophical aspects of probabilistic seismic hazard analysis (PSHA): a critical review. *Natural Hazards*, 117: 1193-1212 <https://doi.org/10.1007/s11069-023-05901-6>.

7. Annex

7.1. Site effects terms for observation sites considered in this study

Table 1. Description of the site effects term for every EDF exploitation site considered in this study. The table includes the average shear-wave site velocity of the upper 30m (V_{s30}), the SSI filtering coefficient (K_{SSI}), and the free field coefficient (K_{soil}). The values inside the parenthesis corresponds to the standard deviation of the SSI filtering coefficient.

Site	V_{s30} (m/s)	k_{SSI}	K_{soil}
Belleville	315	2.01(0.78)	1.5
Blayais	340	1.55(0.33)	1.5
Bugey	690	1.24(0.22)	1.35
Cattenom	800	1.22(0.26)	1
Chinon	1020	1.37(0.33)	1
Chooz	960	1.24(0.43)	1
Cruas	1320	1.28(0.28)	1
Civaux	1510	1.51(0.39)	1
Dampierre	495	1.63(0.44)	1.35
Flamanville	1900	1.05(0.06)	1
Fessenheim	580	1.32(0.26)	1.35
Golfech	445	1.33(0.27)	1.35
Gravelines	290	1.66(0.40)	1.5
Nogent	765	1.26(0.28)	1.35
Paluel	640	1.45(0.39)	1.35
Penly	980	1.22(0.22)	1
Saint-Alban	280	1.89(0.70)	1.5
Saint-Laurent	475	1.63(0.41)	1.35
Tricastin	390	1.82(0.47)	1.5

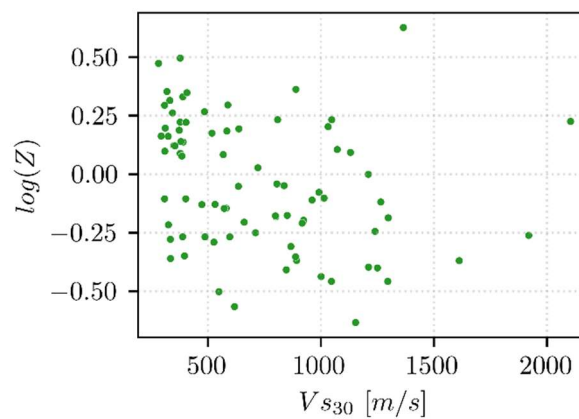


Figure 15. Site effect term for every EPOS-France observation site considered in this study as a function of average shear-wave site velocity of the upper 30m (V_{s30}), (Traversa, et al., 2020).

7.2. Goodness-of-fit of the Pólya distribution to the cumulative number of exceedances

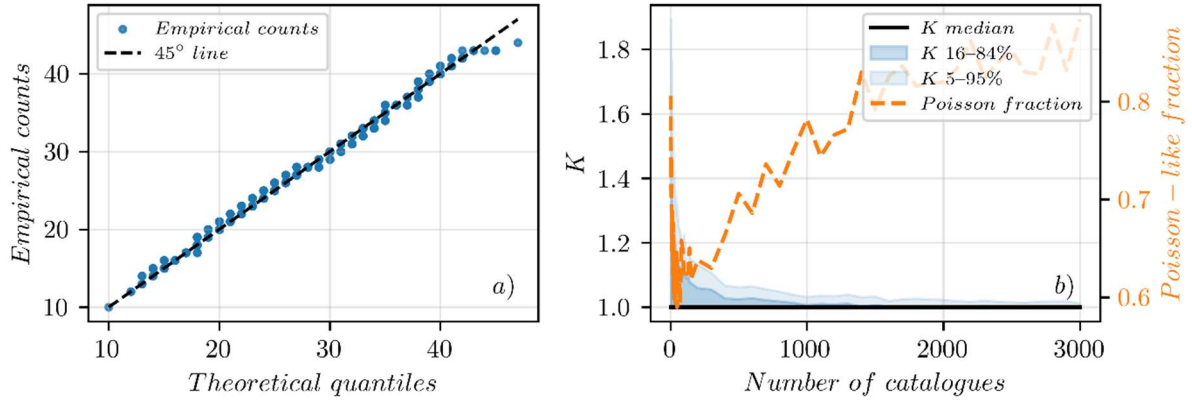


Figure 16. (a) -Goodness-of-fit of simulated correlated exceedance counts to the Pólya distribution for simulations conducted according to branch #7 of the EDF source model combined with the A17 ground-motion model using a QQ plot, and for the set of stations available in July 1980. (b) - Stability of the fitted distribution as a function of the number of realizations (catalogs). Shaded areas represent different percentiles of the estimated correlation coefficient obtained through bootstrapping. A secondary axis shows the fraction of simulated catalogs for which independence—represented by a Poisson distribution—was observed (dashed line).

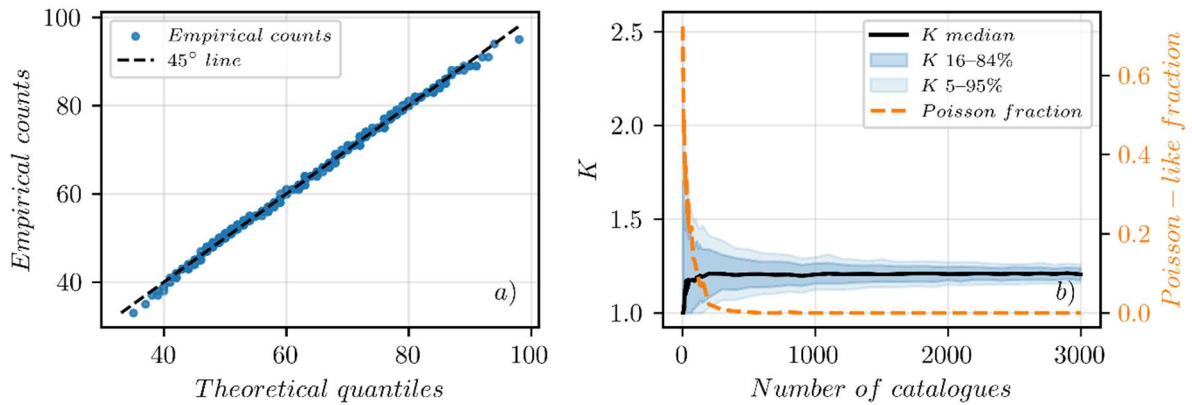


Figure 17. (a) -Goodness-of-fit of simulated correlated exceedance counts to the Pólya distribution for simulations conducted according to branch #7 of the EDF source model combined with the A17 ground-motion model using a QQ plot, and for the set of stations available in July 1995. (b) - Stability of the fitted distribution as a function of the number of realizations (catalogs). Shaded areas represent different percentiles of the estimated correlation coefficient obtained through bootstrapping. A secondary axis shows the fraction of simulated catalogs for which independence—represented by a Poisson distribution—was observed (dashed line).

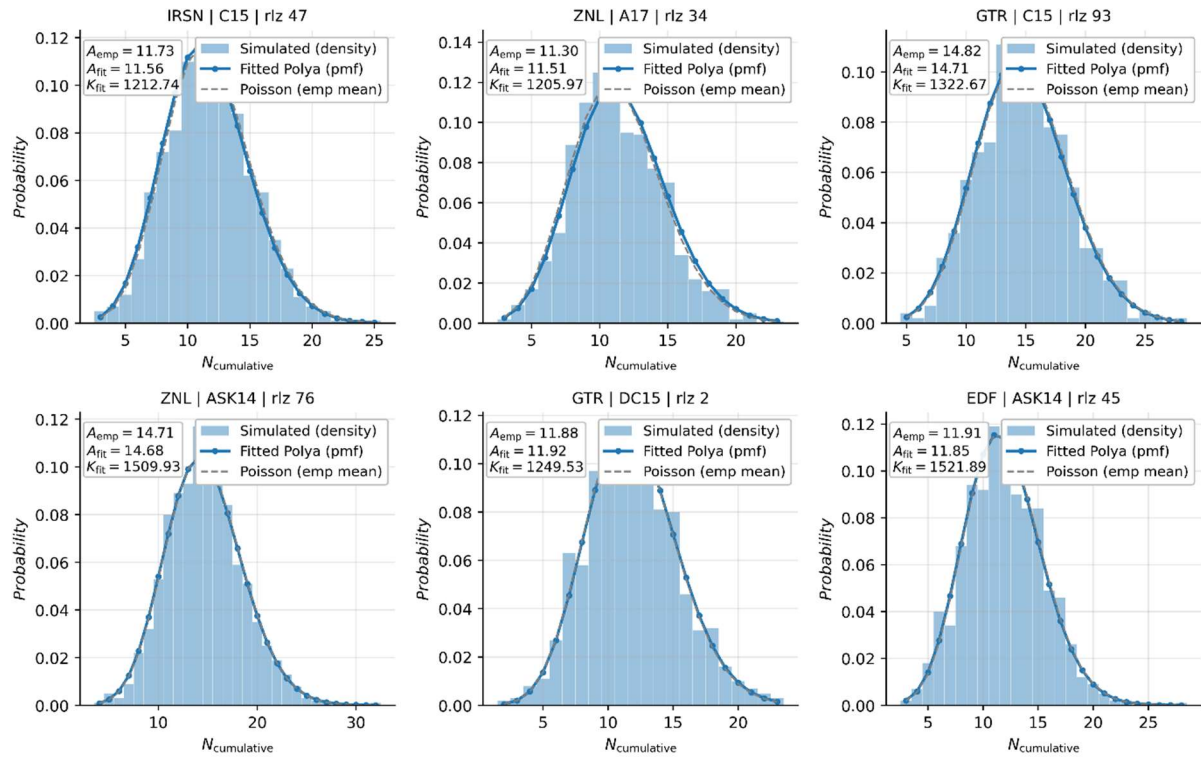


Figure 18. Fitting of the cumulative count of exceedances for an IML of 0.01g to the Pólya distribution for a series of randomly selected branches

7.3. Drouet et al., 2020 evaluation considering EDF observation sites

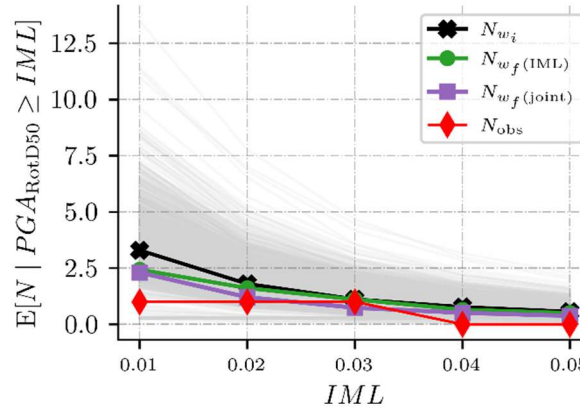


Figure 19. Estimated number of exceedances for different IMLs across EDF observation sites. Gray lines represent the estimates from each branch of the logic tree. The black line with “x” markers show the mean prediction using the original (prior) weights. The green line with circular markers corresponds to the combination using updated (posterior) weights obtained separately for each IML, while the violet line with square markers represents the combination using updated weights derived from the joint probability distribution across all IMLs. The observed number of exceedances is shown in red with diamond markers.