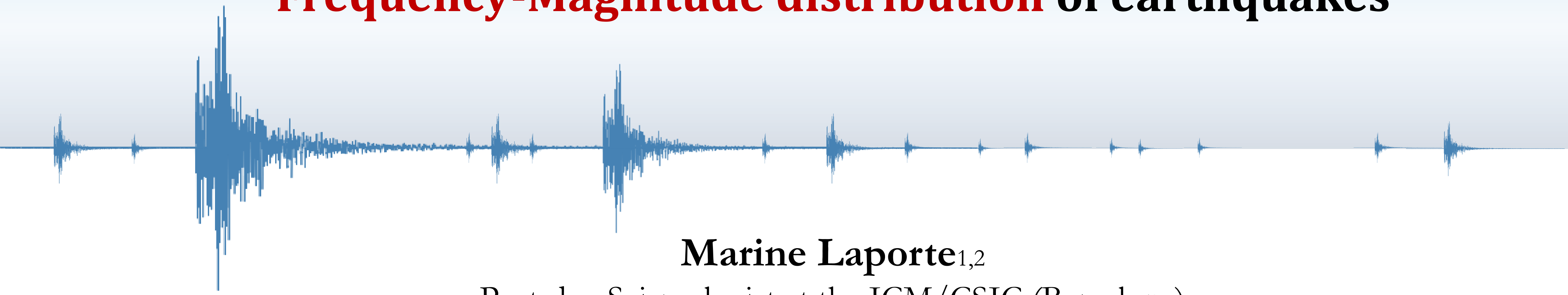
On-going projects about the

Frequency-Magnitude distribution of earthquakes



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In collaboration with :

Stéphanie Durand[2], **Blandine Gardonio**[3], **Thomas Bodin**[1,2]

David Marsan[3], **Cyrielle Colin**[2], **Guillaume Daniel**[4], **Pierre Arroucau**[4]

b-GRASP

Co-funded by the European Union

1-Instituto de Ciencias del Mar (ICM) Consejo Superior de Investicacion Cientifica, Barcelona
2- Laboratoire de Geologie de Lyon - Terre, Planetes, Environnement. –Univ. Lyon1- ENS de Lyon
3- Institut des Sciences de la Terre, Universite Savoie Mont Blanc
4 – Electricité de France (EDF), Aix en Provence

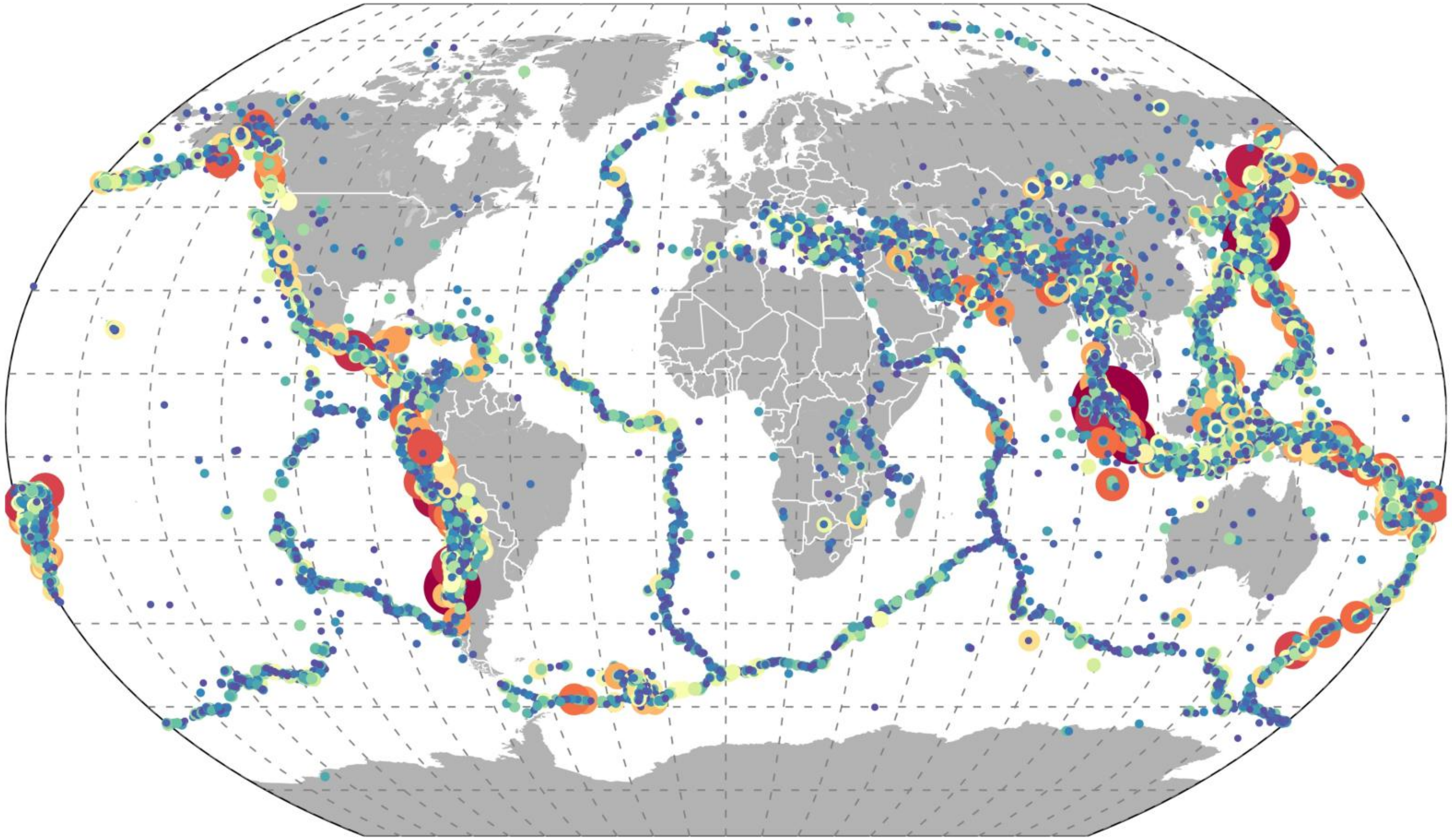
Institut de Ciències del Mar

LGL TPE

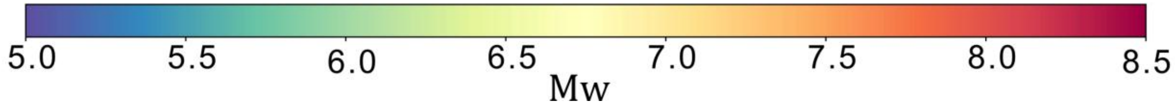
Frequency-magnitude distribution

Global scale

How many earthquakes above magnitude 7 can we expect every year ?



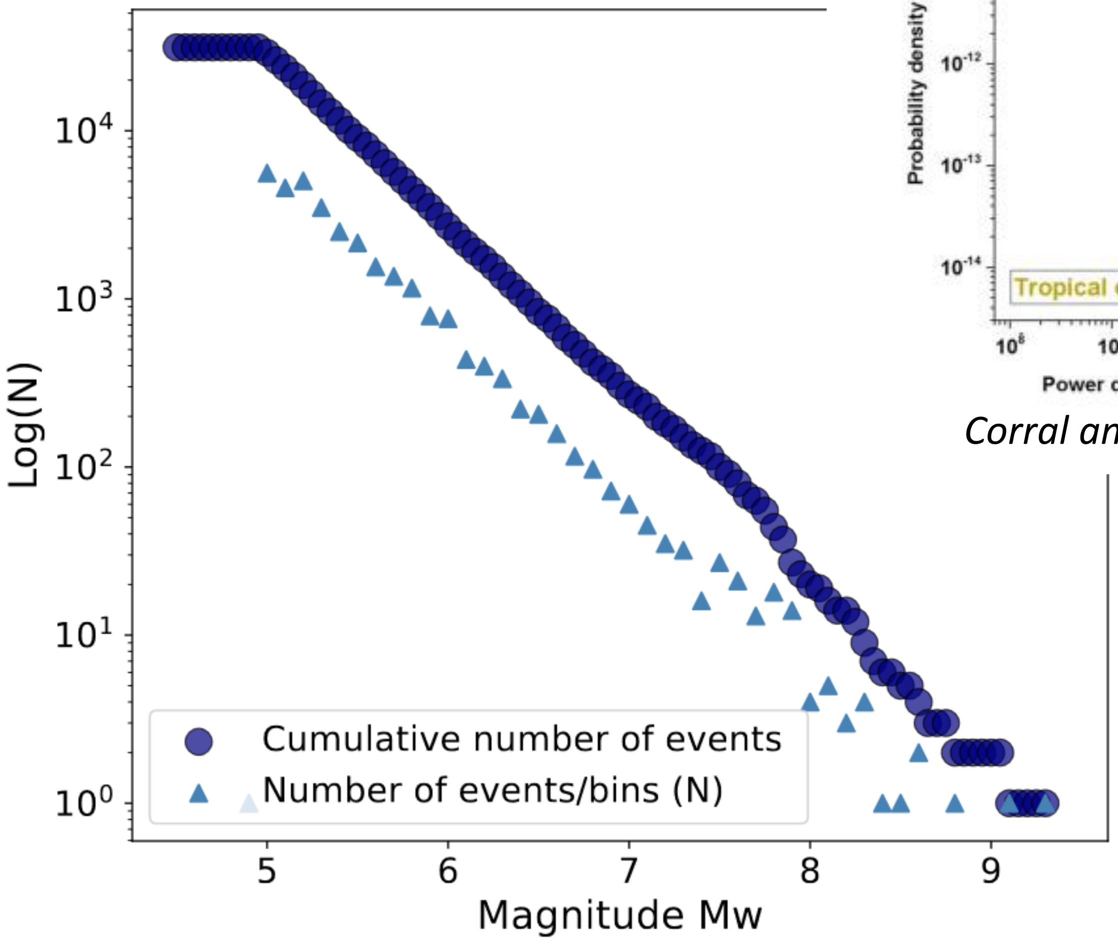
ISC-GEM - Global Intrumental Earthquake Catalog (1999-2019)
(Storchak et al., 2013; 2015; Di Giacomo et al., 2018)

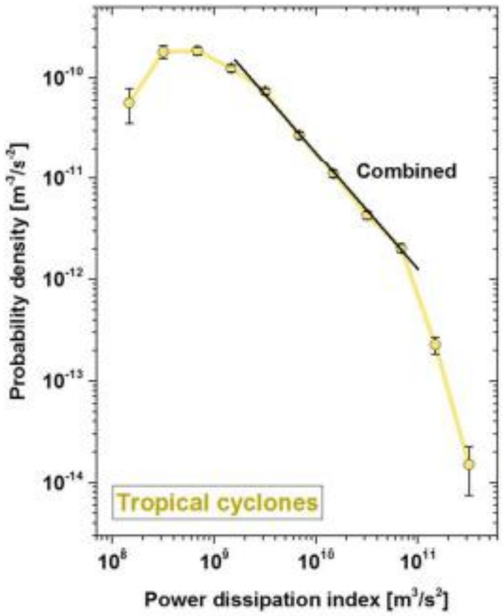


General Frequency-Size Distribution

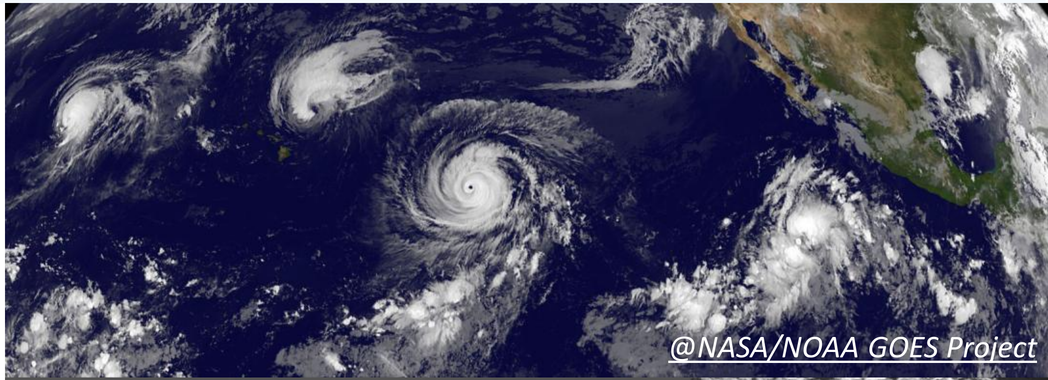
Frequency-Magnitude distribut

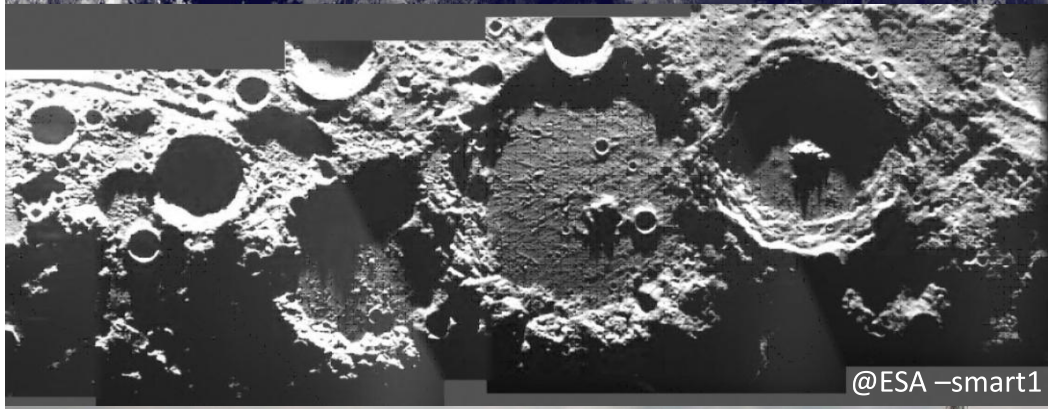
ISC-GEM Global Instrumental Earthqu





Corral and Gonzalez., 2019

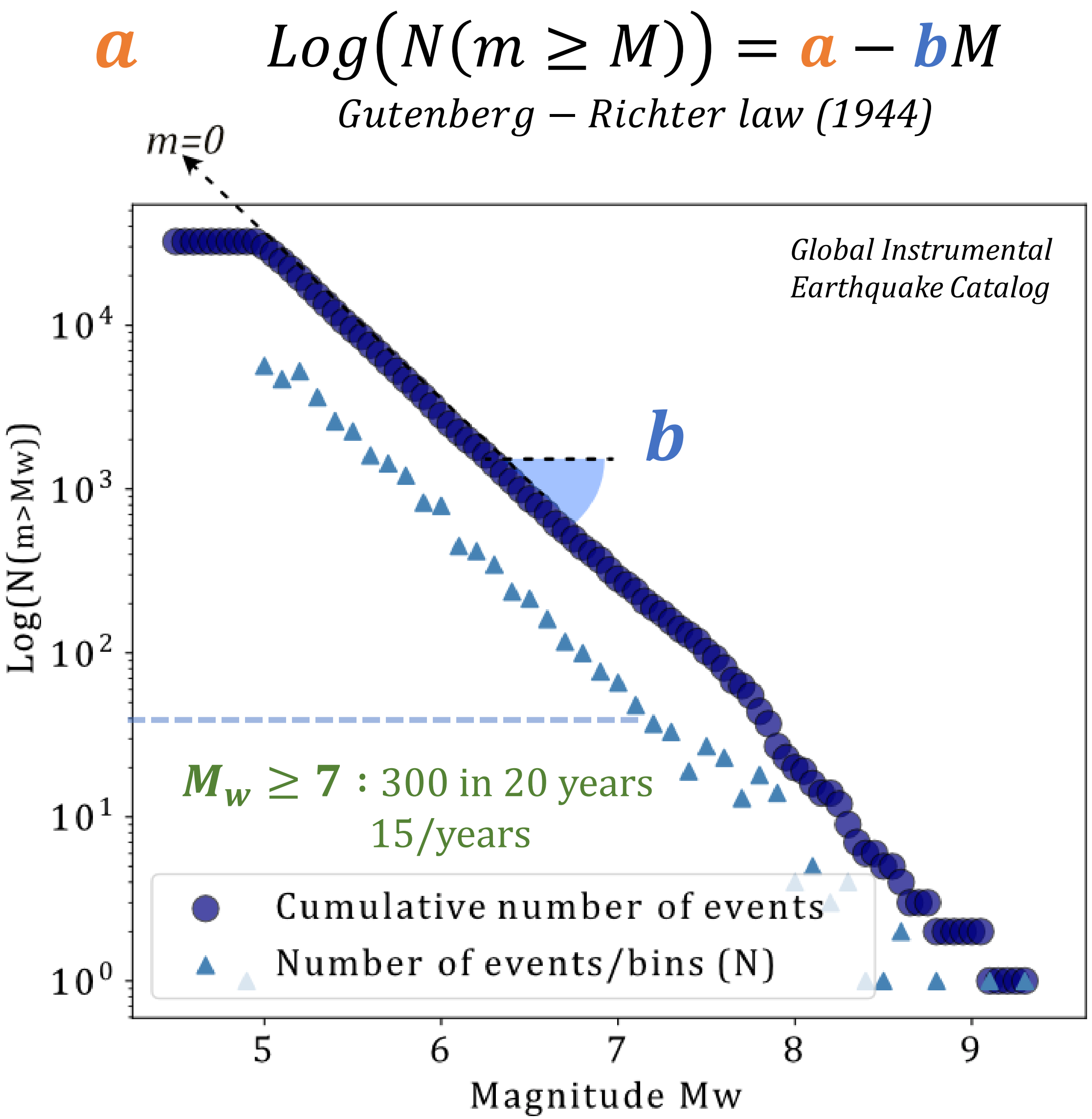




Frequency-Magnitude Distribution

$$Log(N(m \geq M)) = a - bM$$

Gutenberg – Richter law (1944)



a = *Seismic rate*

$$a_{annual} = \log(\frac{N_{m>0}}{\Delta T_{years}})$$

France → *Nepal*

Frequency-Magnitude Distribution

a

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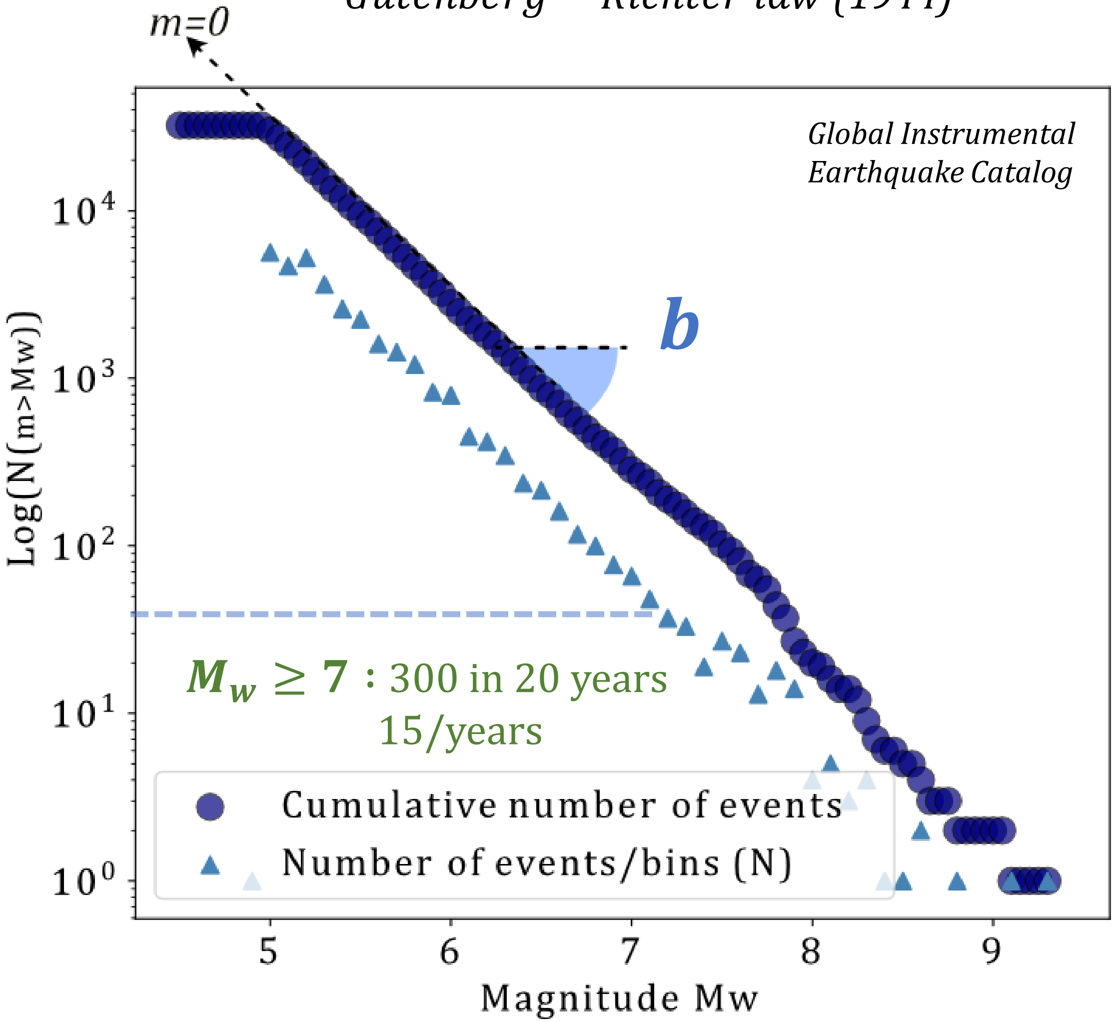
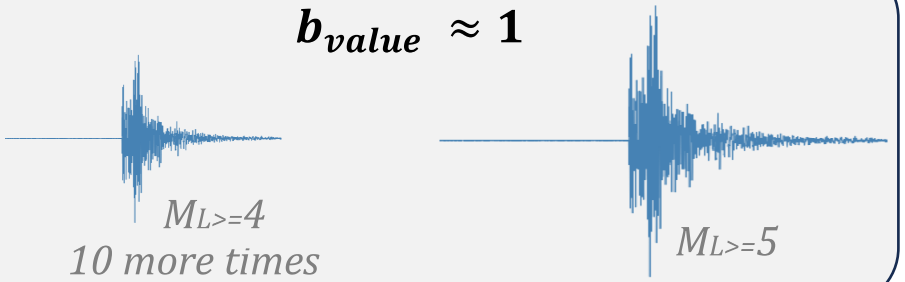


$b_{value} \approx 1$



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Mw7.8 Gorkha earthquake Nepal

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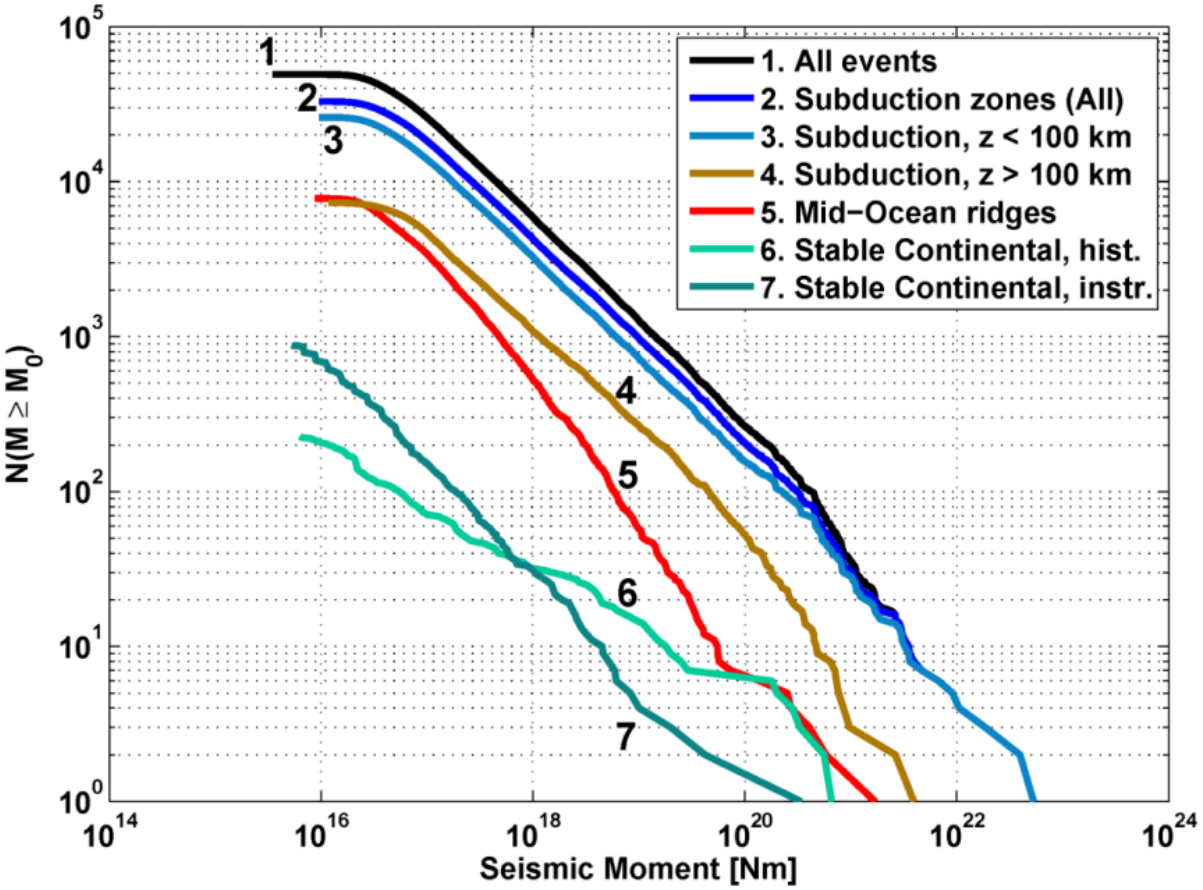
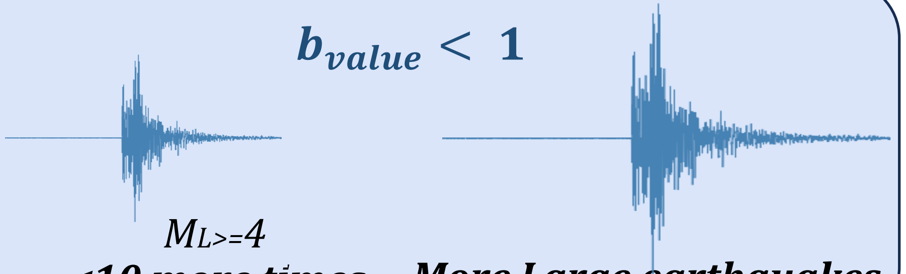


Knapmeyer et al., 2023

$b_{value} \approx 1$

$b_{value} < 1$



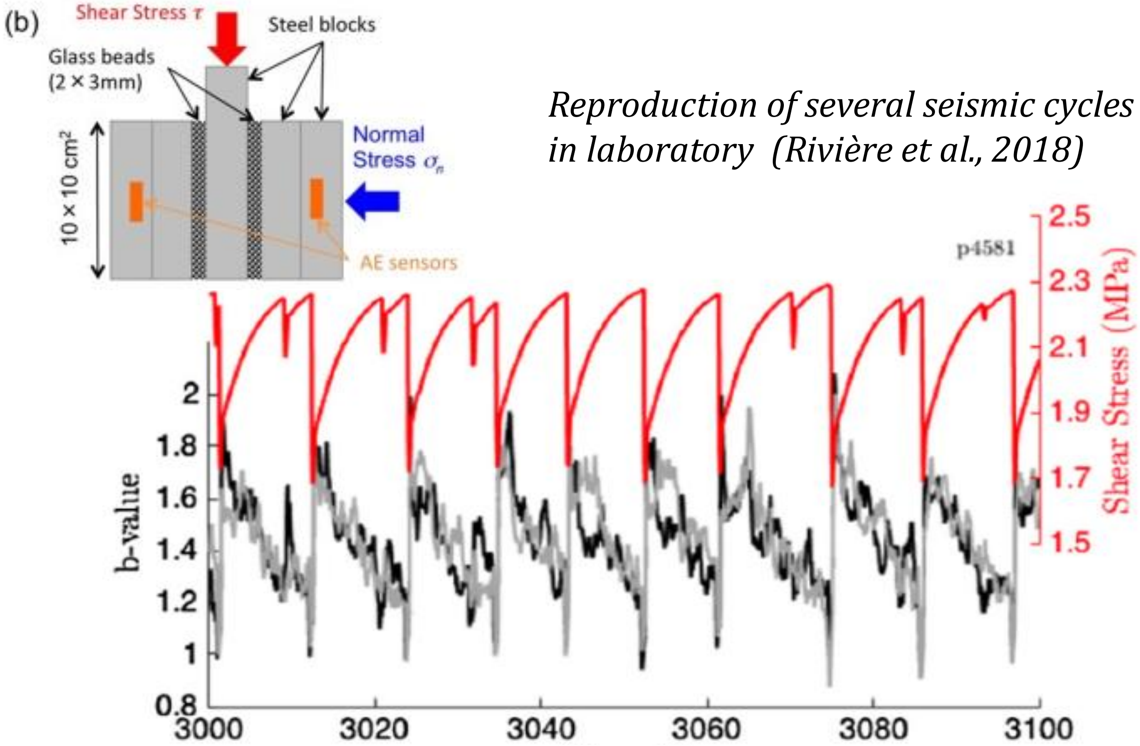
Mw7.8 Gorkha earthquake Nepal

Motivations / Impacts

I1. b_{value} can be used as **a proxy of the tectonic stress**

An observation from experimental studies…

b_{value} decreases with increasing stress (e.g. Scholz., 1968)



Reproduction of several seismic cycles in laboratory (Rivière et al., 2018)

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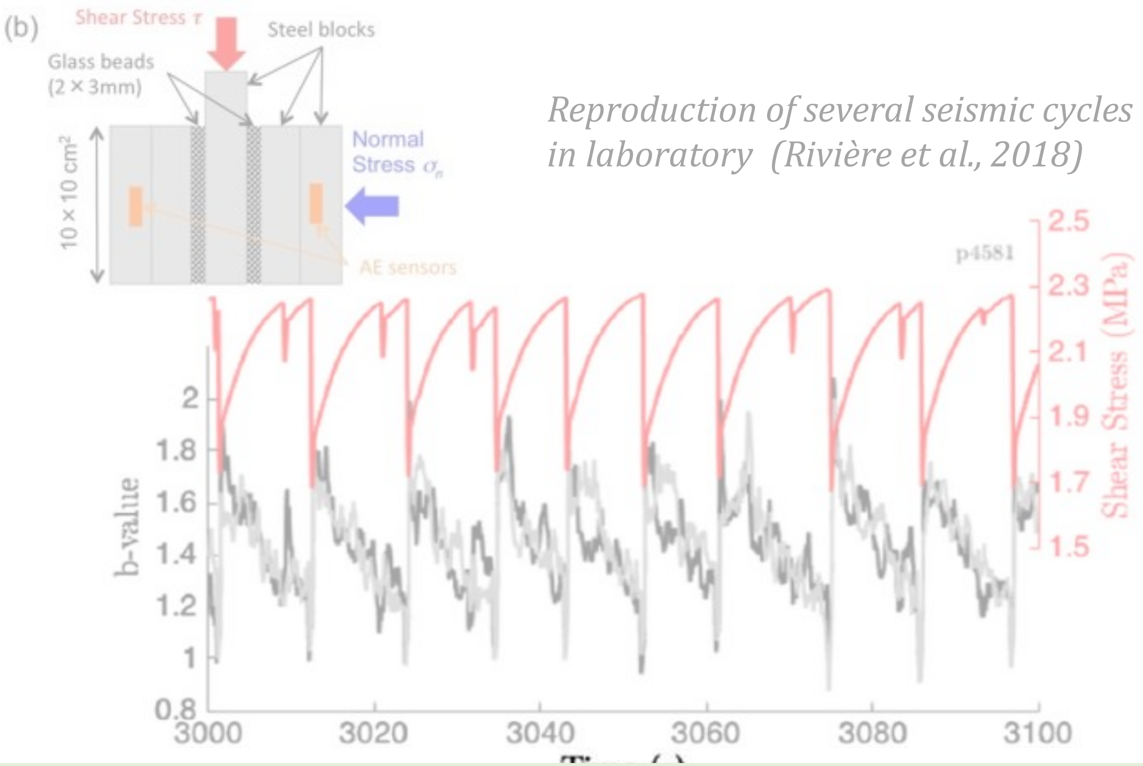
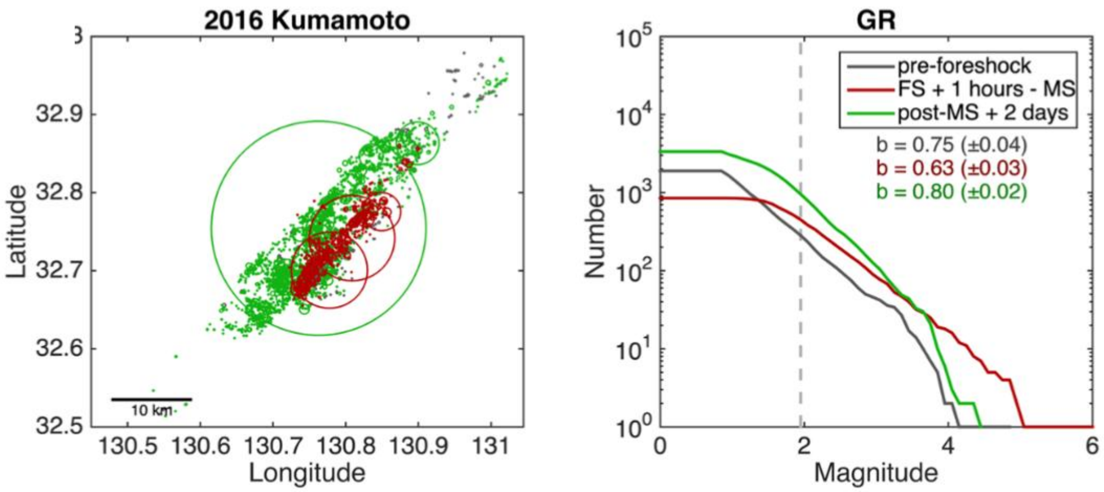


Reproduction of several seismic cycles in laboratory (Rivière et al., 2018)

… confirmed with foreshocks sequences !

Kumamoto earthquake Mw7.0 (2016, Japan) – 277 deaths
Foreshock sequence (Mw6.2) – Mainshock sequence

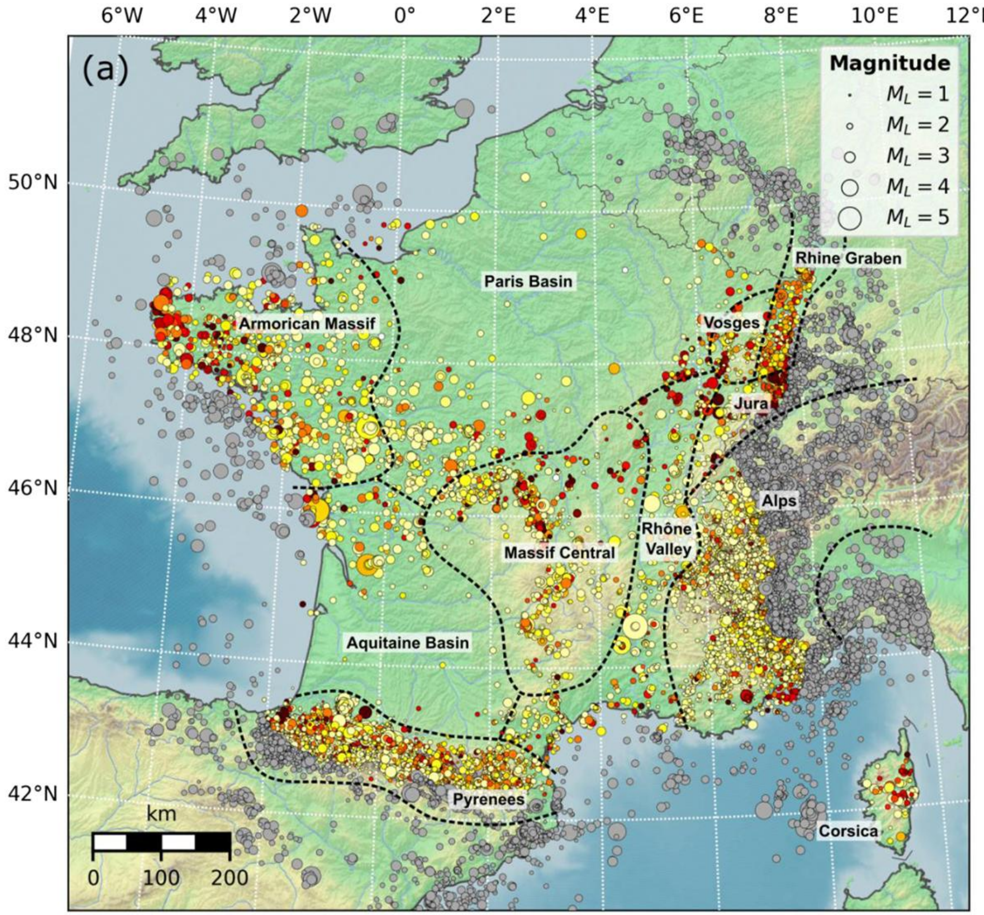
Van der Elst., 2021



We want to estimate in a robust way the temporal evolution of b-value

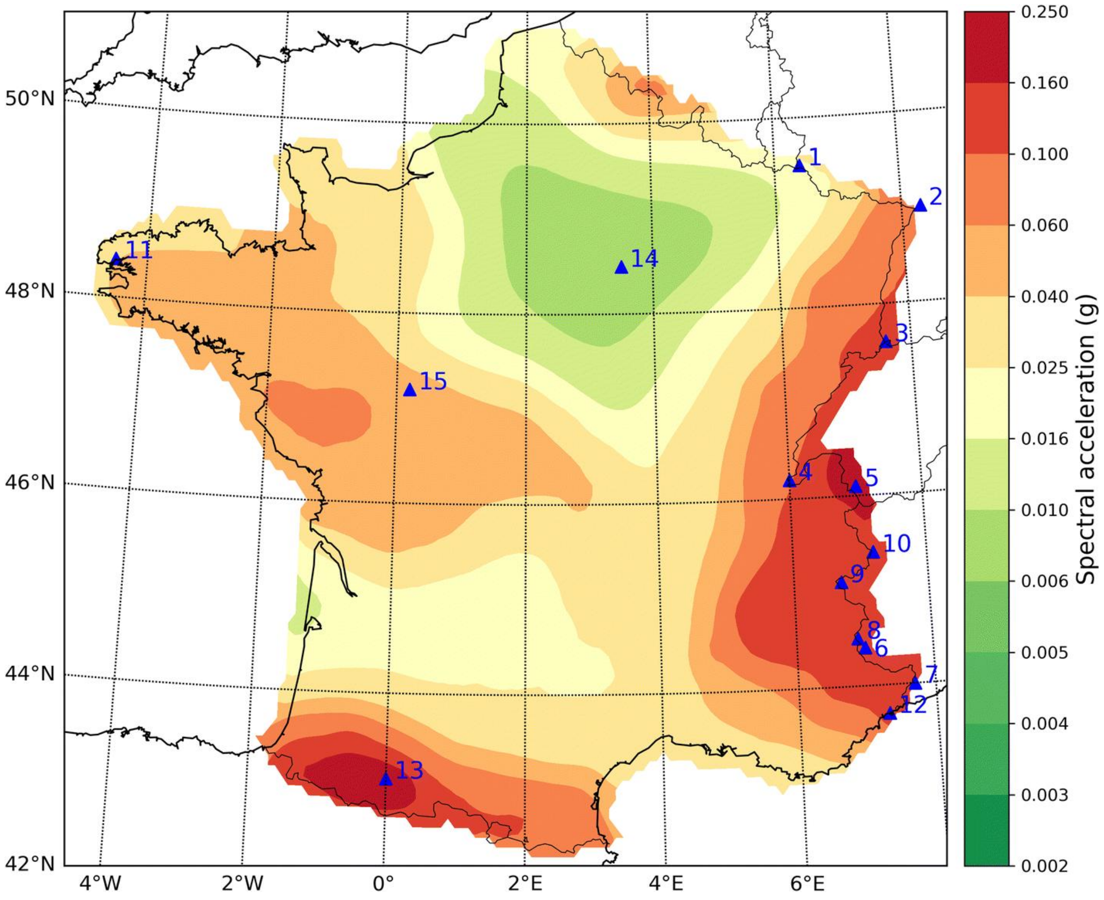
Motivations / Impacts

I2. b_{value} and seismic rates are used in earthquake recurrence models



?

Earthquake recurrence models



We want to estimate in a robust way the spatial distribution of b-values and seismic rates

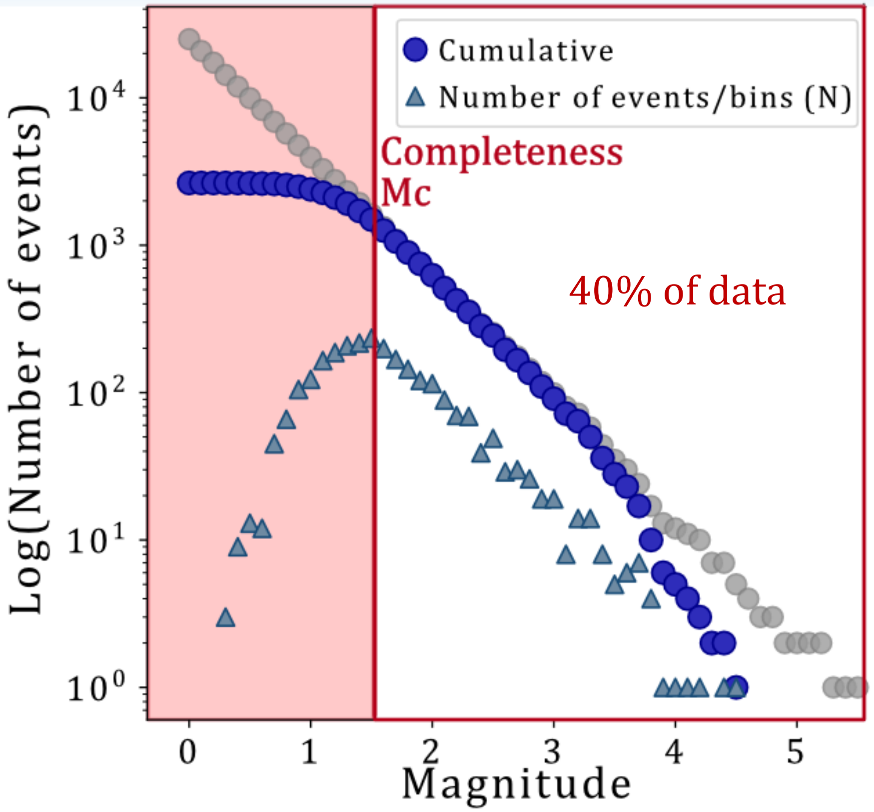
Plan

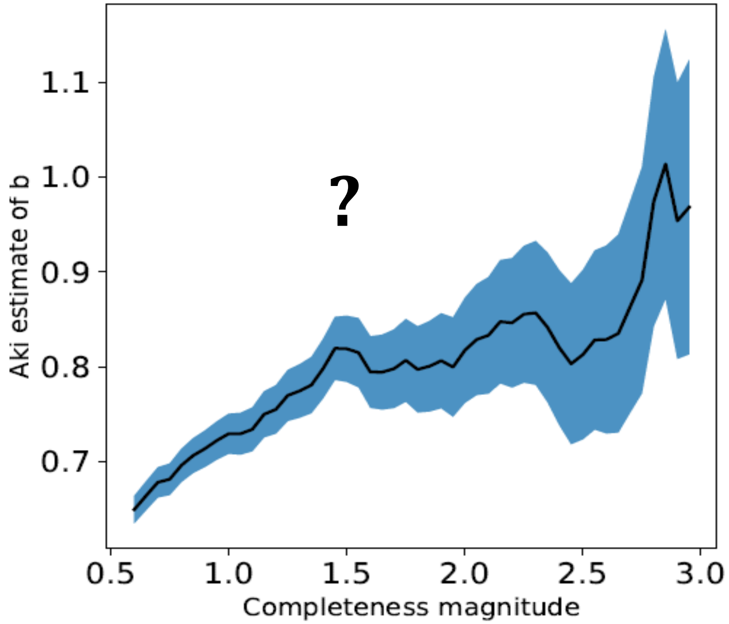
How do we estimate b-value and seismic rates spatial and temporal variations ?

Part 1 : Usual b-value estimators and their limitations

- The frequentist approach of Aki *(1965)*
- Incompleteness sources from long-term to short term
- Usual integration of temporal variations of completeness
- New challenges for earthquake data analysis

Classical approaches can be improved





The Frequentist approach :

finds **b-value** that maximizes the likelihood $P(\boldsymbol{m_{obs}}|\boldsymbol{\beta})$

$$\boldsymbol{\beta} = \frac{\mathbf{1}}{\langle \boldsymbol{m} \rangle - \boldsymbol{M_c}}$$ *Aki (1965)*

$$b_{value} = \frac{\boldsymbol{\beta}}{\log(10)}$$

Issues :

- b-value **depends on the choice of Mc**
- **Seismic rate estimated independantly**

- **Truncate the catalog**
- A **non probabilistic** estimate
- **Mc varies with time and space**

Incompleteness sources (1)

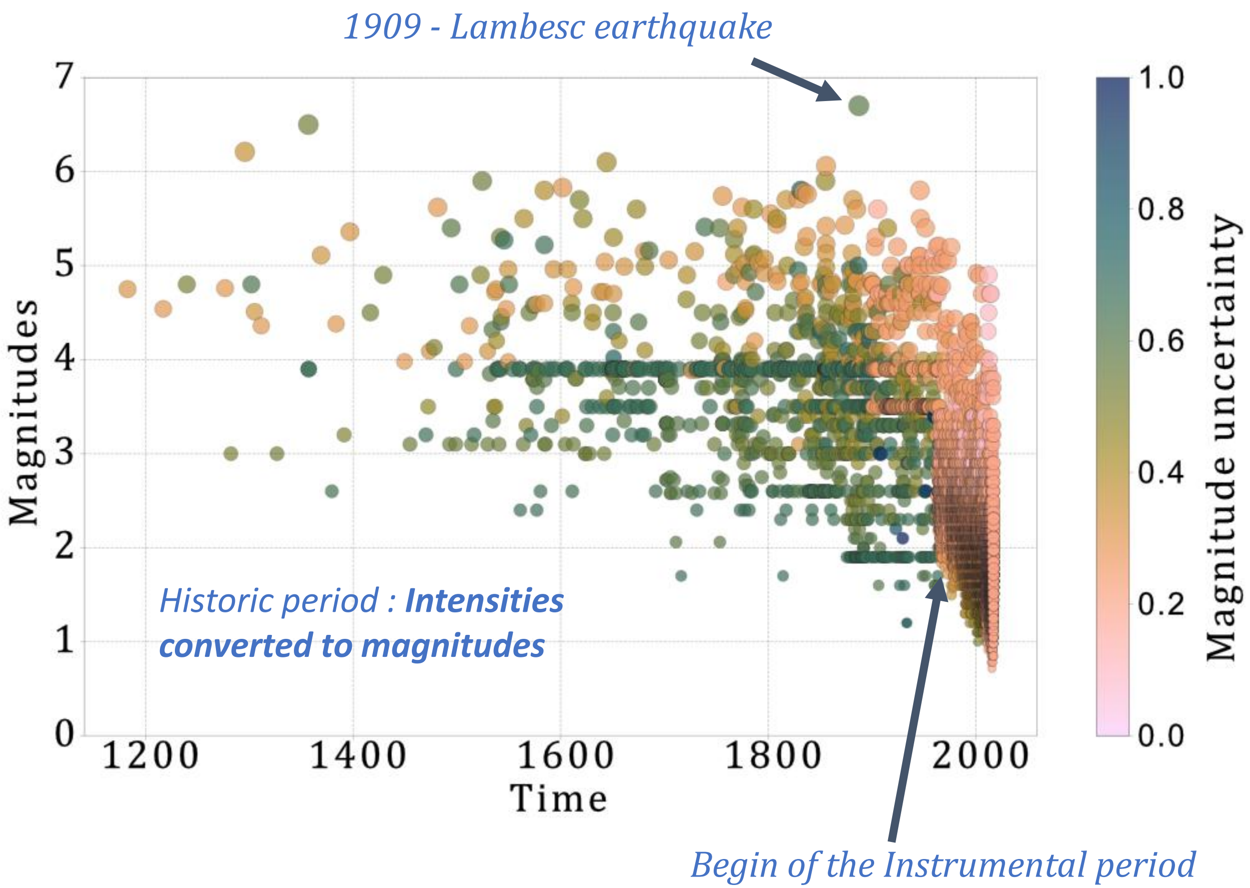
(1) Long-Term Datasets (Historical data)

- **Combination of Historical and Instrumental seismicity**

Large incompleteness during Historical period

Large variations during the Instrumental period

Picture of the 1909 – Lambesc earthquake



Equivalent Magnitude over Time of the French Historical and Instrumental catalog from Drouet (2020) Colin et al. (under review)

Incompleteness sources (1)

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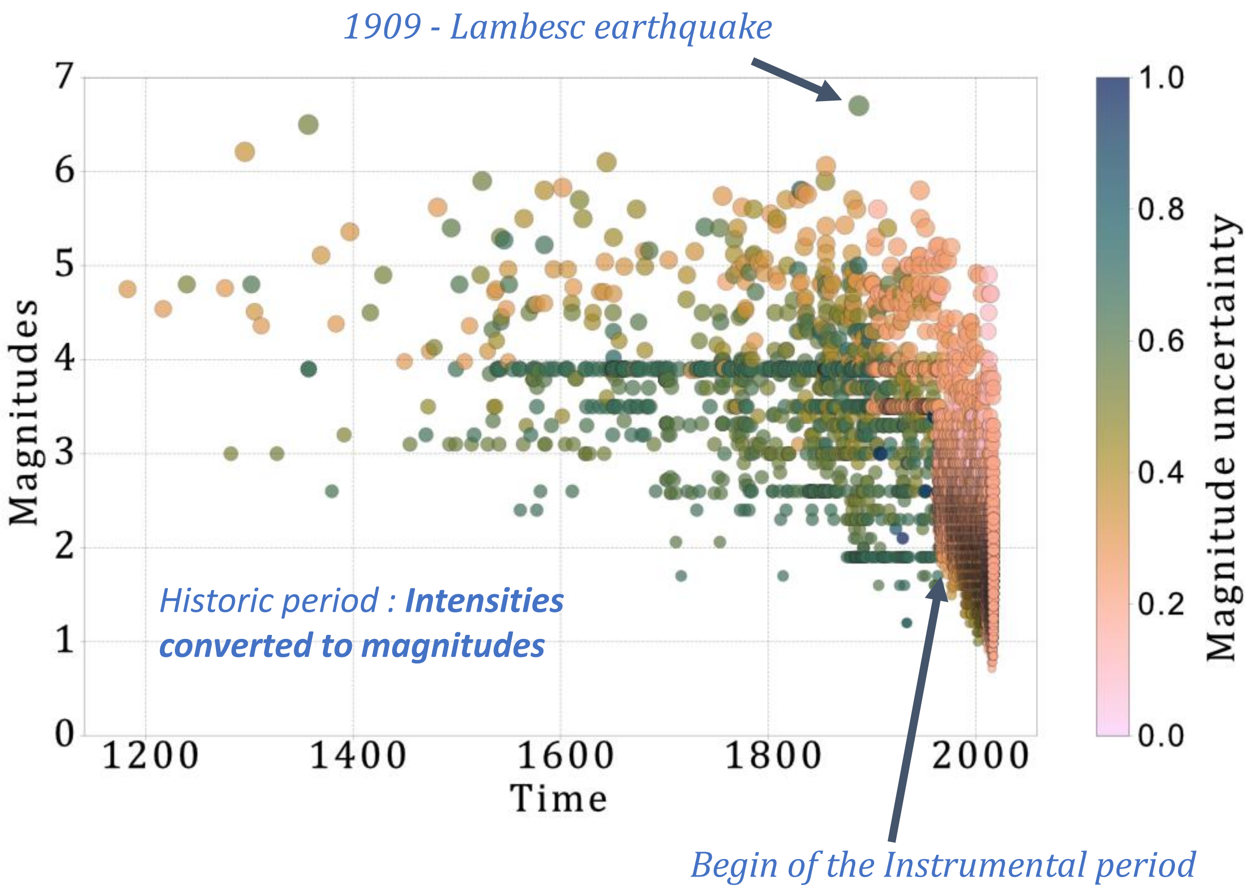
Picture of the 1909 – Lambesc earthquake

PhD Thesis Celine Leblanc

Sigma3 – Starting December 25

Bayesian Inversion of Seismic Intensities for Historical Earthquake Analysis

Supervision: Thomas Bodin (ICM), Pierre Arroucau (EDF)

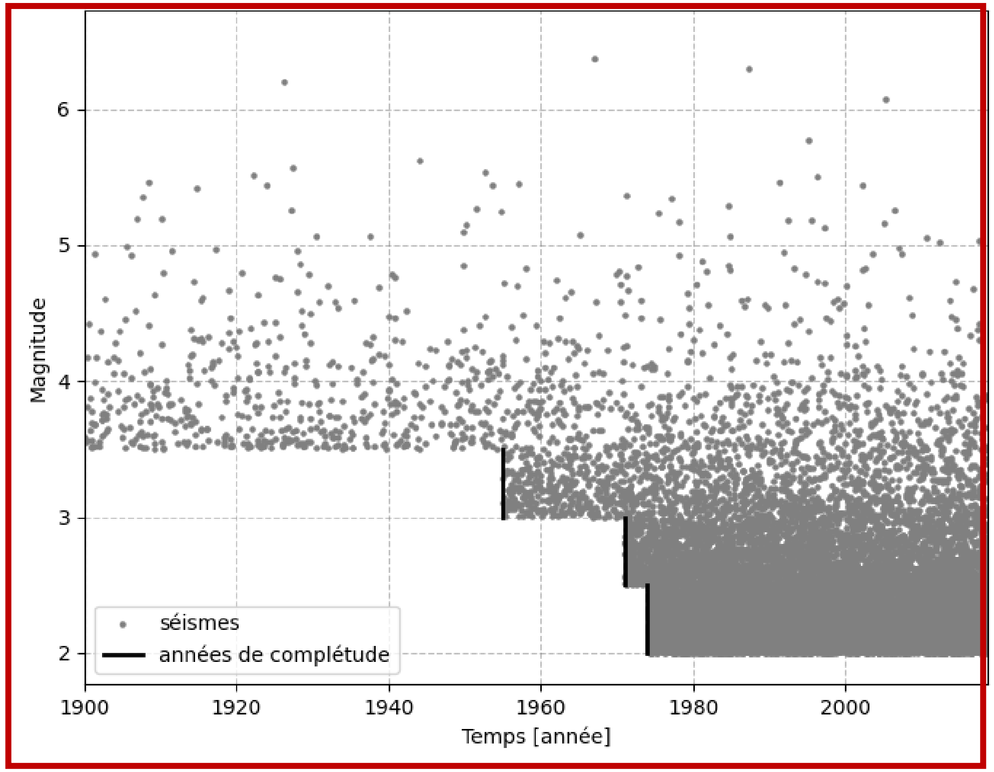


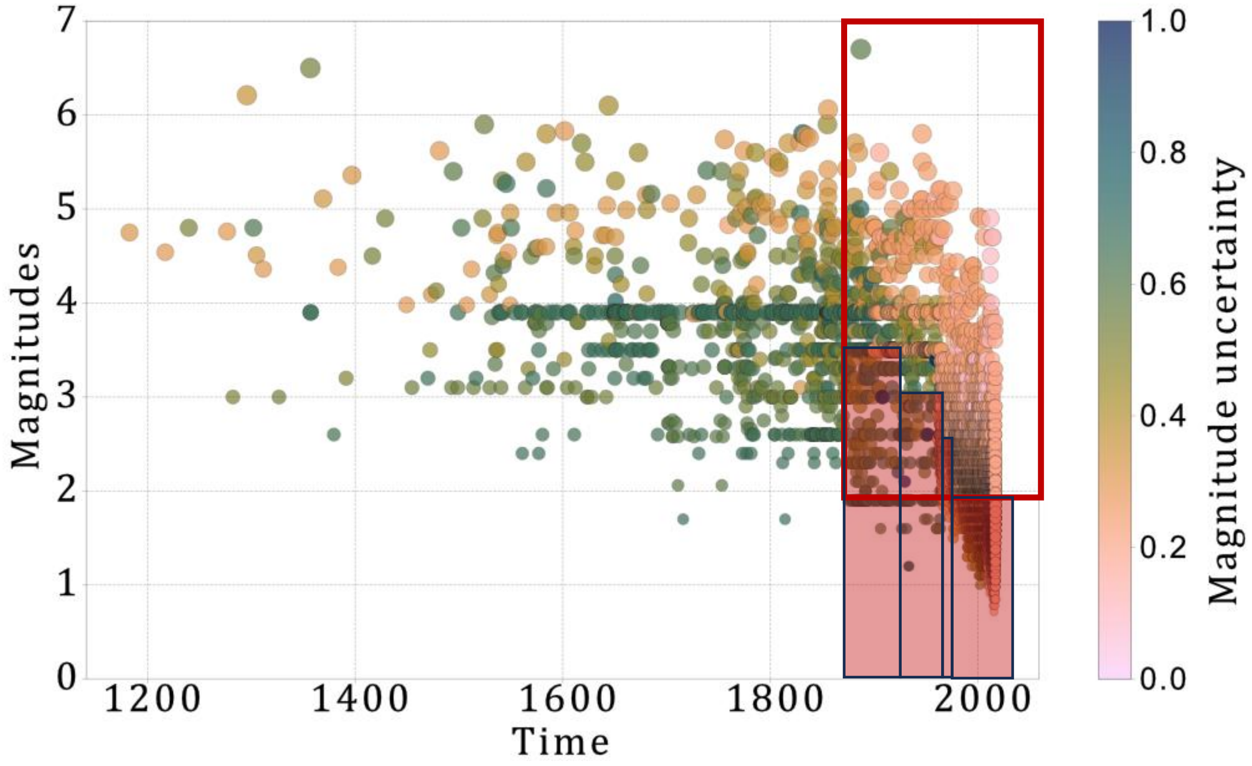
Equivalent Magnitude over Time of the French Historical and Instrumental catalog from Drouet (2020)
Colin et al. (under review)

Capturing temporal changes of incompleteness

Introduction of **periods of completeness** for magnitude intervals (Weichert., 1980)

Truncating the catalog in periods of time





Issues :

Same issues as Aki (1965) :

- b-value depends on the arbitrary **choice of completeness periods**
- **Not probabilist**
- Truncated catalog : lost information

Incompleteness sources (2)

(2) Circumstantial incompleteness

- **Momentarily higher seismic noise**

Meteorological

Anthropic (days/nights/week-ends/Covid)

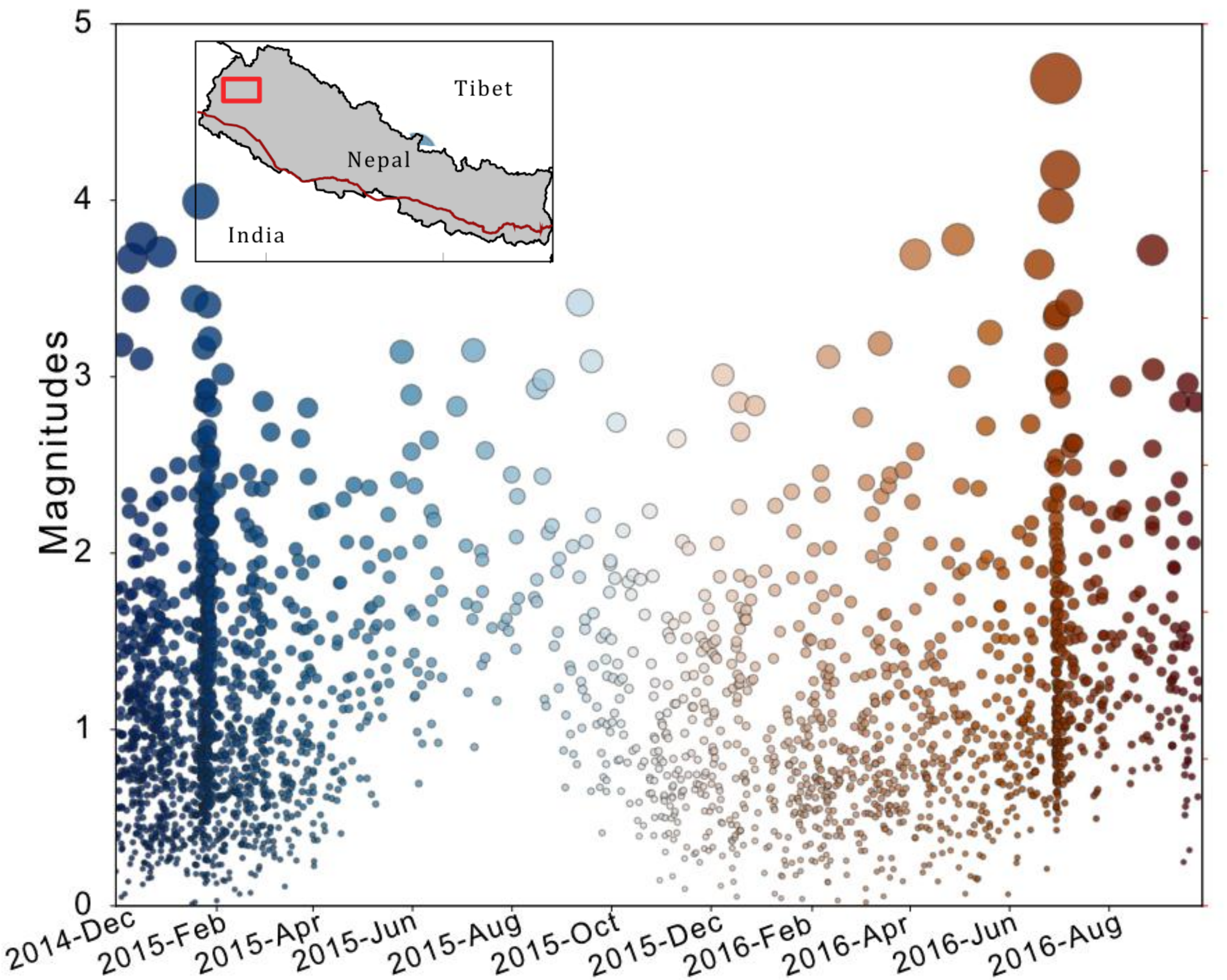
- **Seismic network modifications**

Addition/Loss of seismic stations

Change in the network geometry

Exemple 1 : Far-western Nepal microseismicity
HiKNet network

- **Location :** Far-western Nepal
- **Context :** Temporary seismological experiment
- **Time period :** 2 years (2014-2016)
- **Catalog size :** 2500 earthquakes



Incompleteness sources (3)

(3) Short-term aftershock incompleteness (STAI)

- **Sudden increase of the completeness magnitude**

  Large earthquakes hide smaller ones (mainshocks)

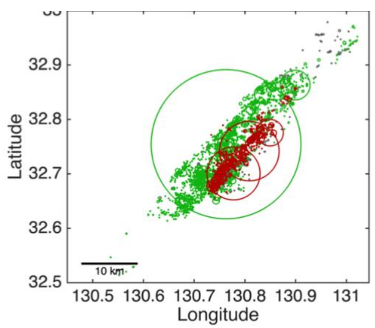
- **Exponential decrease with the aftershock sequence**

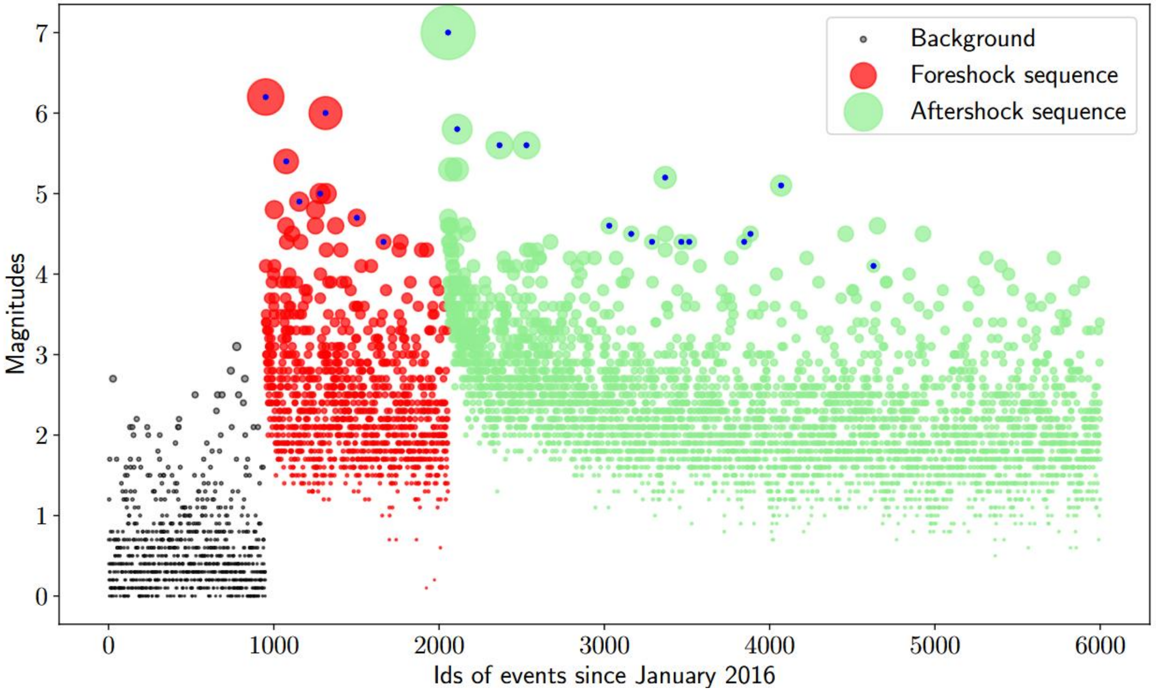
  With the Omori law

  Some formula accounts for STAI (Helmstetter et al., 2006)

Exemple 2 : Kumamoto earthquake sequence

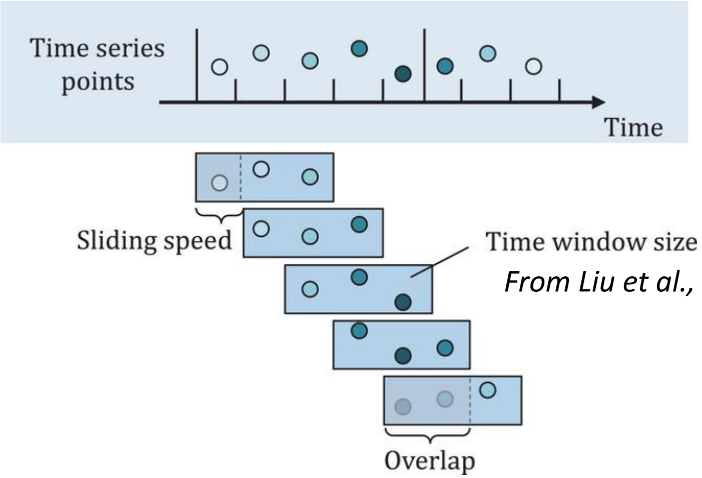
- **Location :** South Japan
- **Context :** Mainshock-aftershock sequence
- **Time period :** 4 months (January-April 2016)
- **Catalog size :** 6000 earthquakes





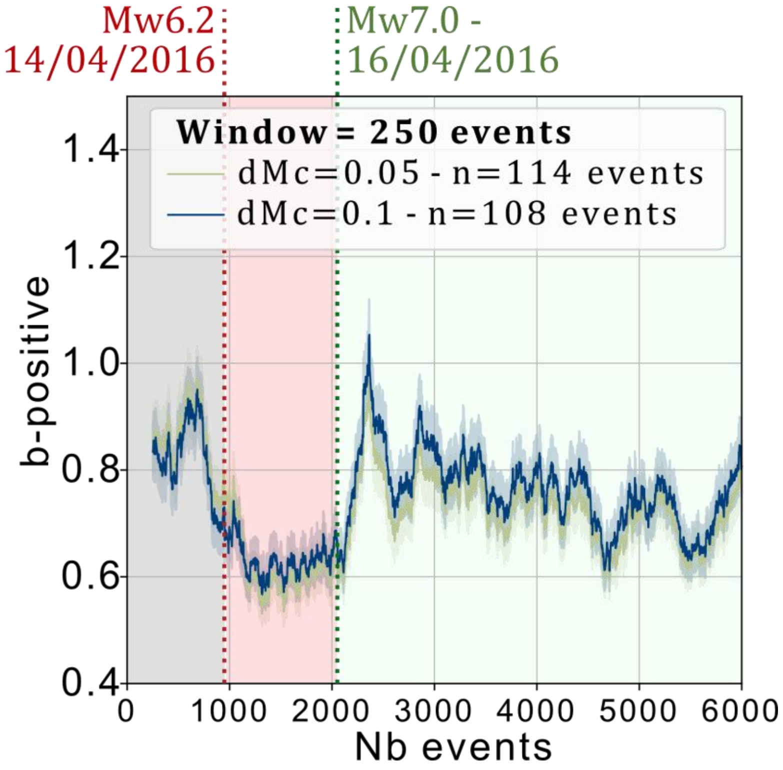
Kumamoto earthquake Mw7.0 (2016, Japan)
Foreshock sequence (Mw6.2) – Mainshock sequence

Capturing temporal changes : using moving windows



From Liu et al., 2022

Window = 250 events



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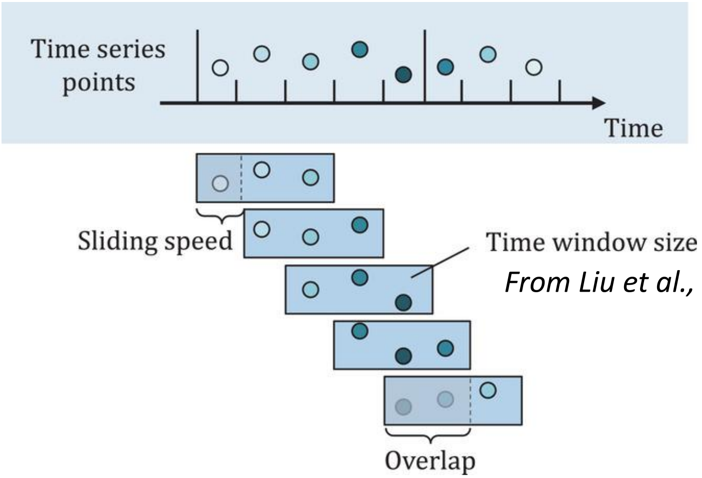

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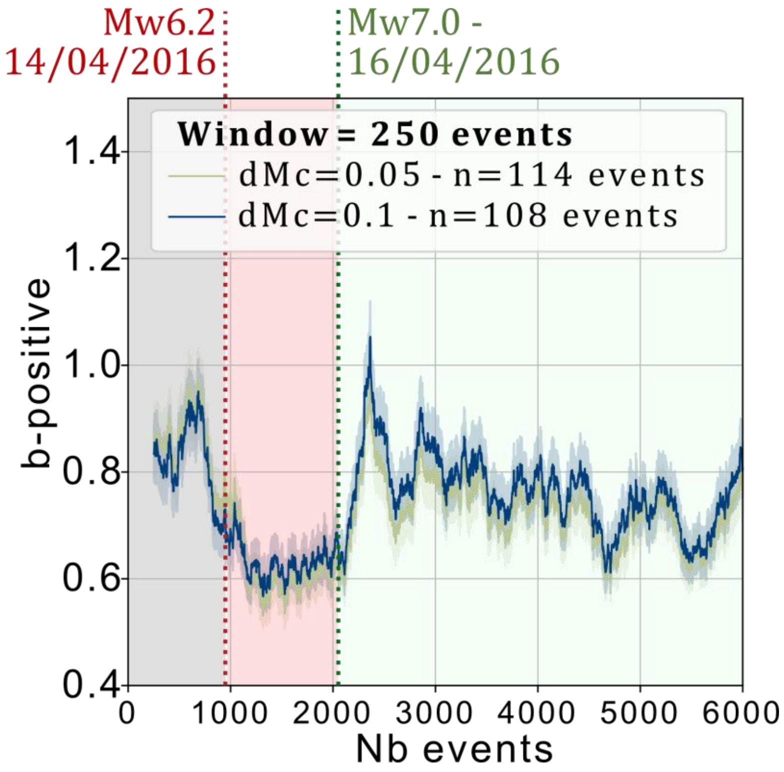
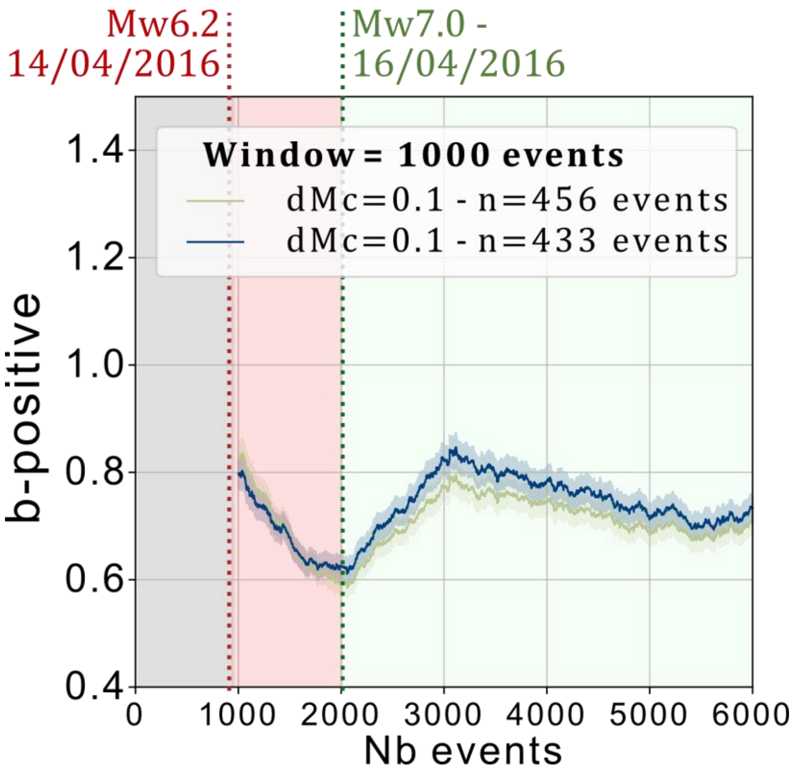


Window = 1000 events



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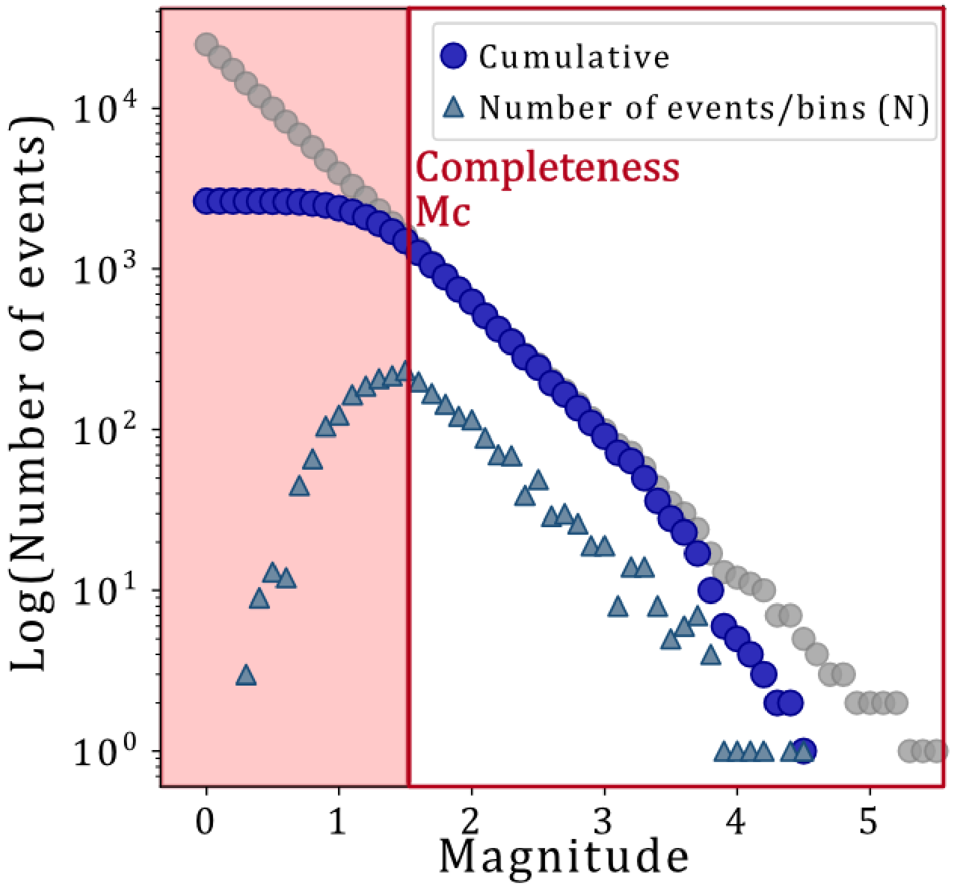
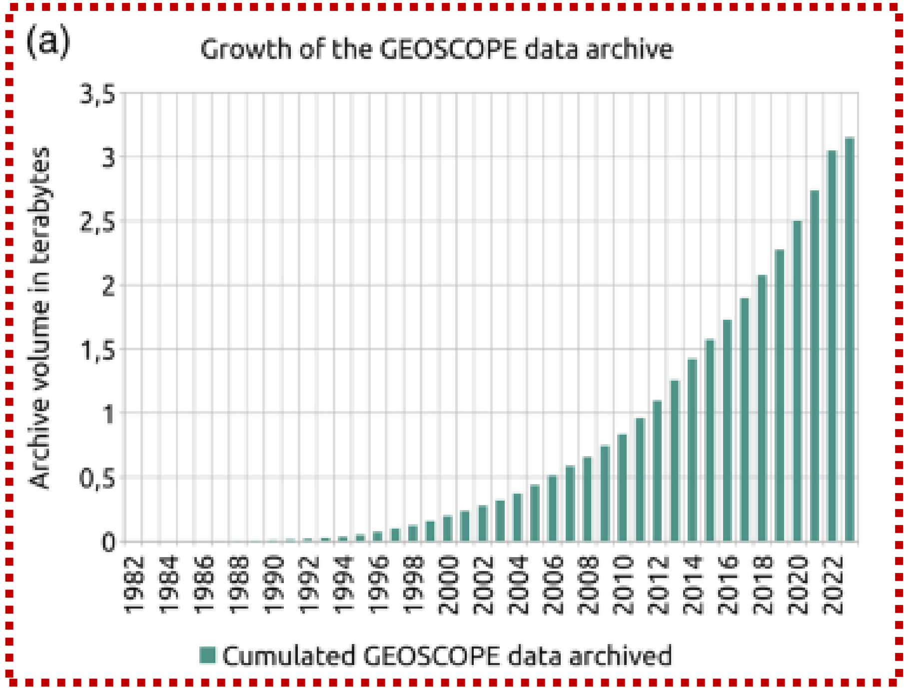
New challenges in exploring large datasets





Leroy et al., 2023

Challenges :

- Use **all the data** available
- **Add complexity** the Gutenberg-Richter model (more parameters)
- Find more robust **inversion of spatial and temporal** variations

Be Bayesian for b-value temporal variations

Part 2 : b-Bayesian : b-value temporal variations for non truncated catalogs

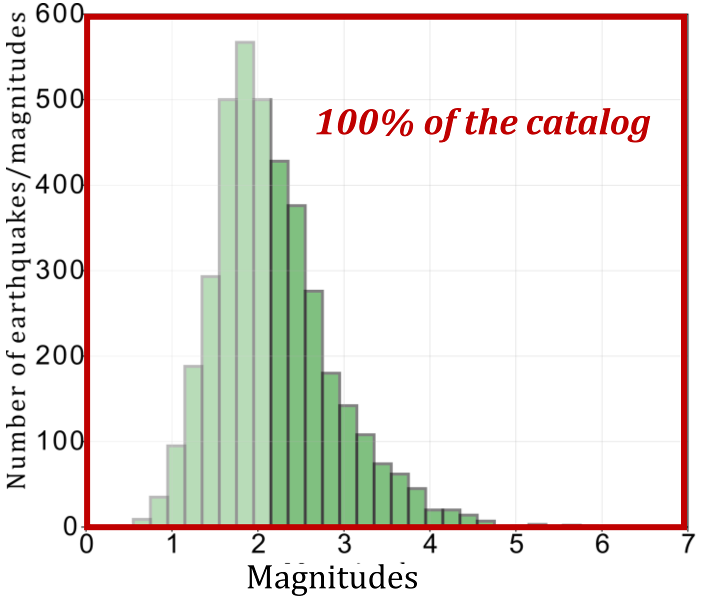
- (1) Conjoint inversion of b-value and detectability (an equivalent to Mc)
- (2) Using a Probabilistic framework
- (3) Inversion of temporal changes with a fully Bayesian approach

New probabilistic approaches for b_{value} estimate without truncating the earthquake catalog

(1) Conjoint inversion of b-value and detectability

Probability of **observation**

Likelihood



We want a method that uses all the data available (Laporte et al., 2025)

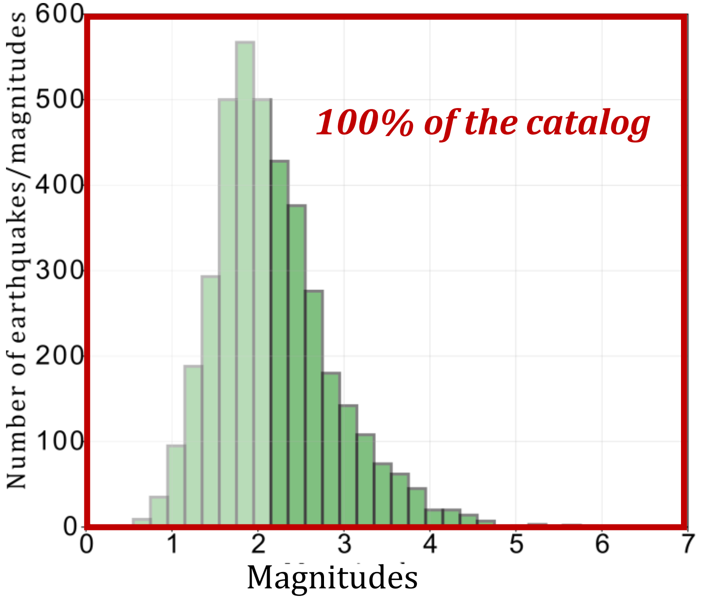
(1) Conjoint inversion of b-value and detectability

Probability of **observation**

Likelihood

Probability of **occurence**

Gutenberg-Richter (1944)



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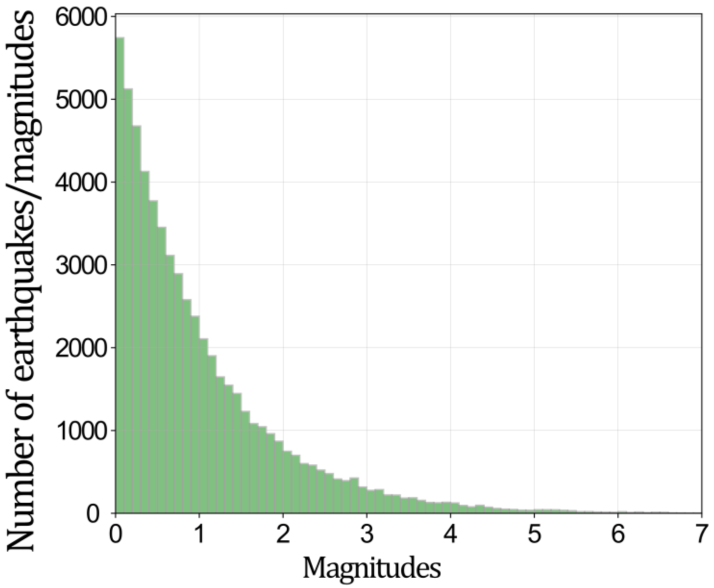
$$N(m) \propto e^{-\beta m}$$



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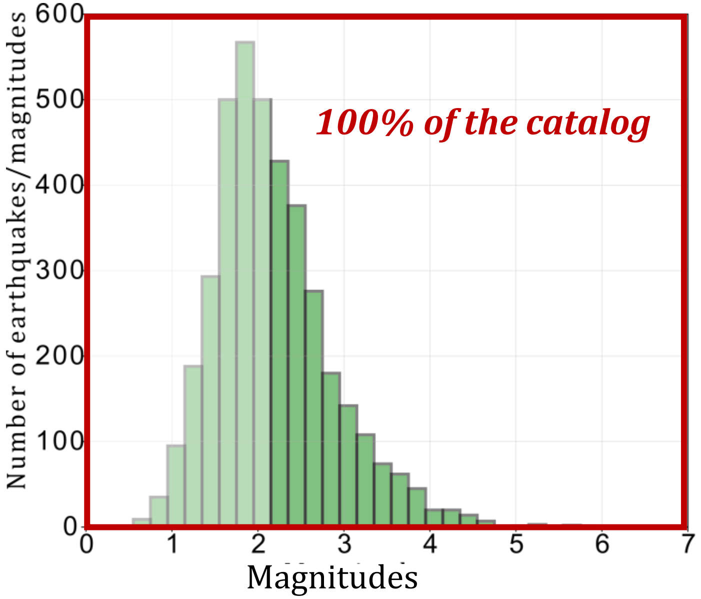

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Probability of **occurence**

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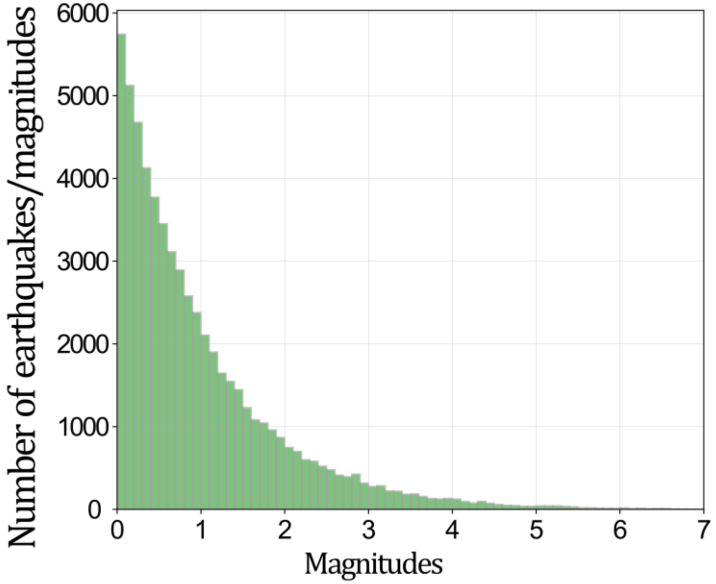

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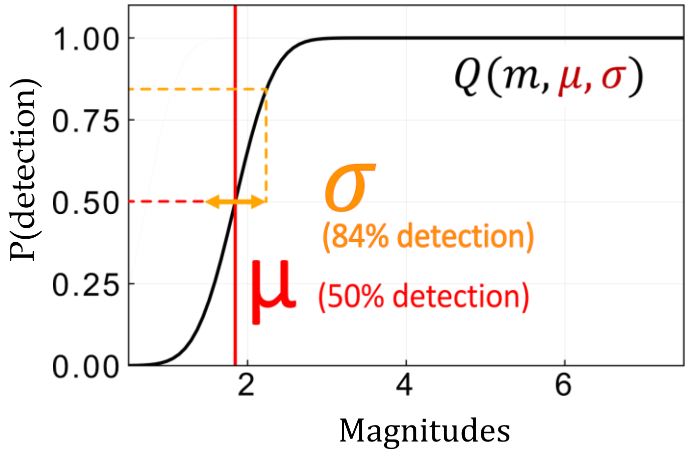
Probability of **detection**

Ogata and Katsura (1993)

$$Q(m) = \frac{1}{2} + \frac{1}{2} erf\left(\frac{m-\mu}{\sqrt{2}\,\sigma}\right)$$



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(1) Conjoint inversion of b-value and detectability

Probability of **observation**
Likelihood

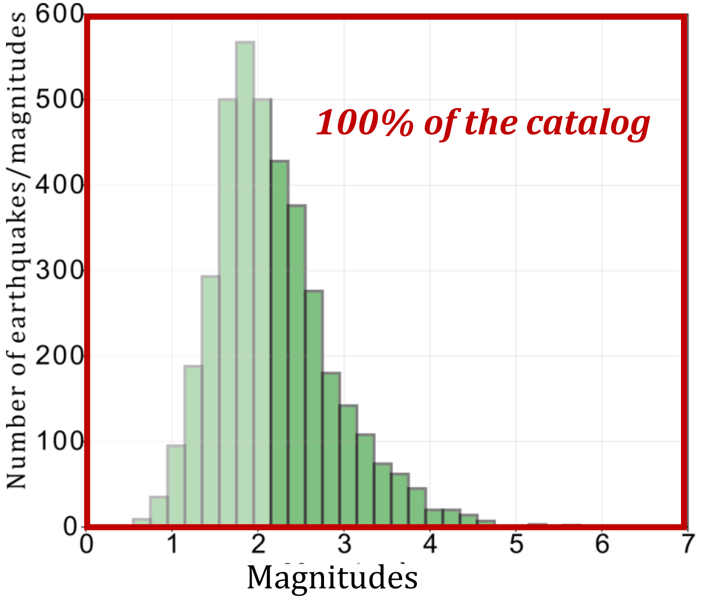
Probability of **occurence**
Gutenberg-Richter (1944)

Probability of **detection**
Ogata and Katsura (1993)

$$P(\boldsymbol{m_{obs}}|\boldsymbol{\beta},\boldsymbol{\mu},\boldsymbol{\sigma}) = \boldsymbol{\beta}^N e^{-N\boldsymbol{\beta}\langle \boldsymbol{m_i}\rangle} \times \mathrm{P}(\mathrm{Q(m)}|\boldsymbol{\mu},\boldsymbol{\sigma})$$

$$N(m) \propto e^{-\boldsymbol{\beta m}}$$

$$Q(m) = \frac{1}{2} + \frac{1}{2} erf\left(\frac{\boldsymbol{m-\mu}}{\sqrt{2}\,\boldsymbol{\sigma}}\right)$$



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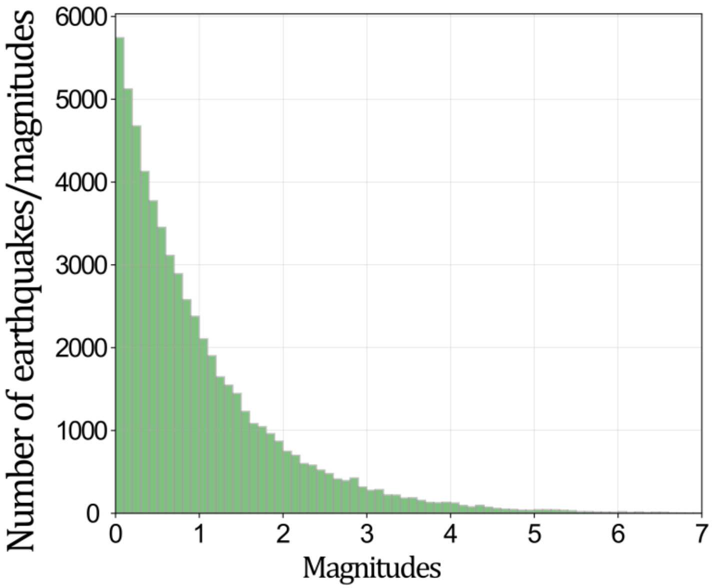

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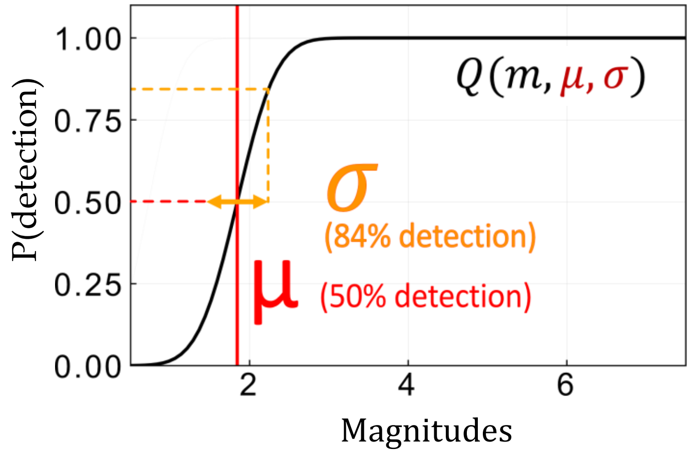

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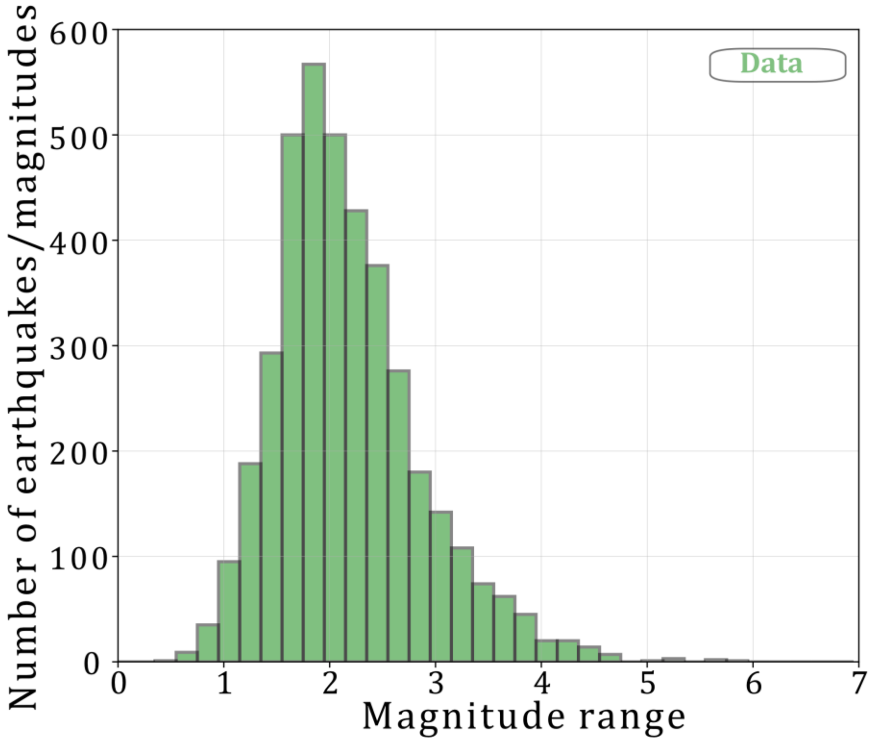
(2) Using a Probabilistic Approach

Synthetic Frequency-magnitude distribution



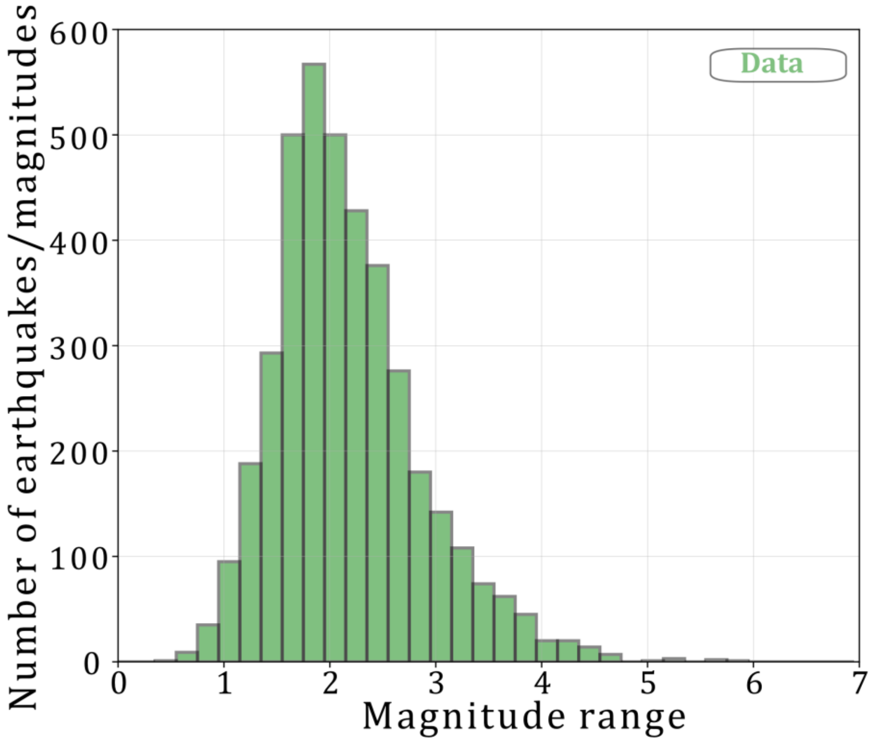
A Bayesian Approach

Posterior		Likelihood		Prior
$P(\mathrm{Model}\|m_{obs})$	$\propto$	$P(m_{obs}\|Model)$	$\times$	$P(Model)$

Model : (3 dim)
{b-value , μ ,σ}

(2) Using a Probabilistic Approach

Synthetic Frequency-magnitude distribution



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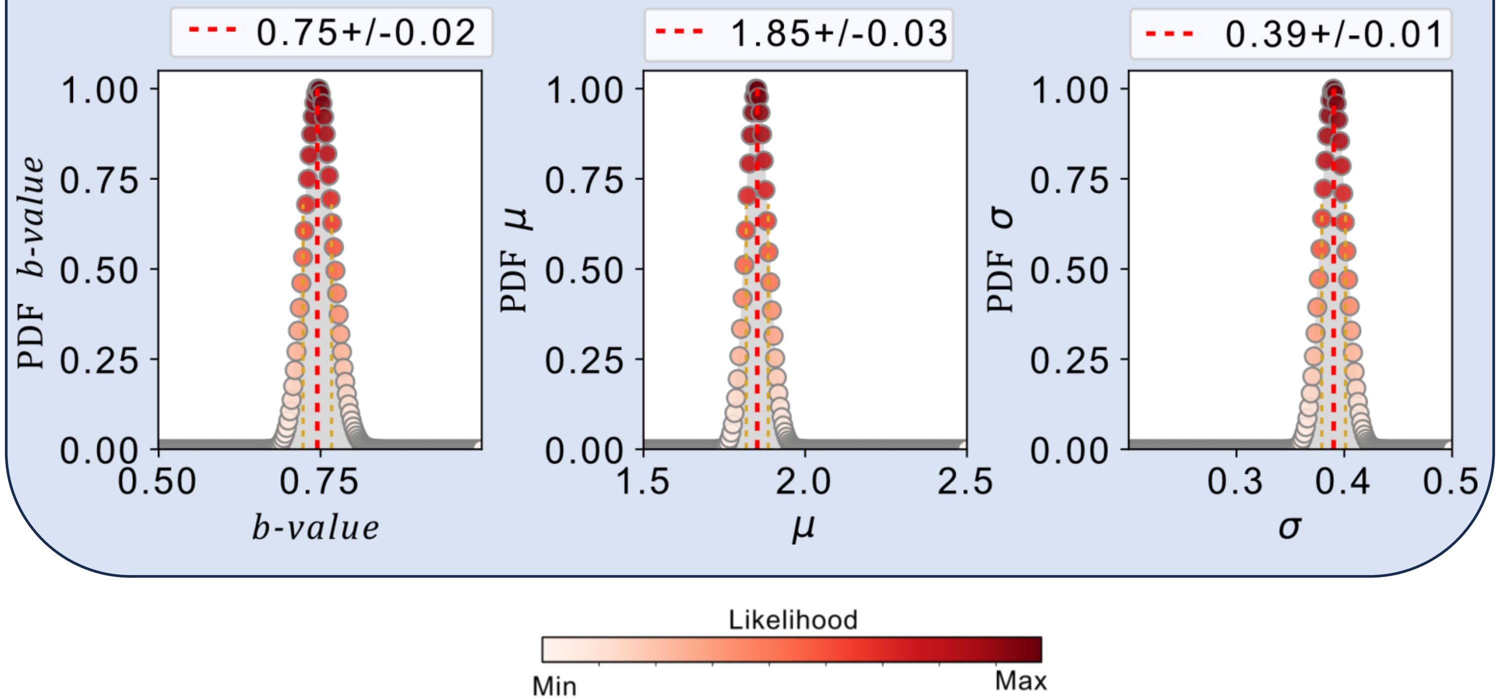
3D - Probability Density Function



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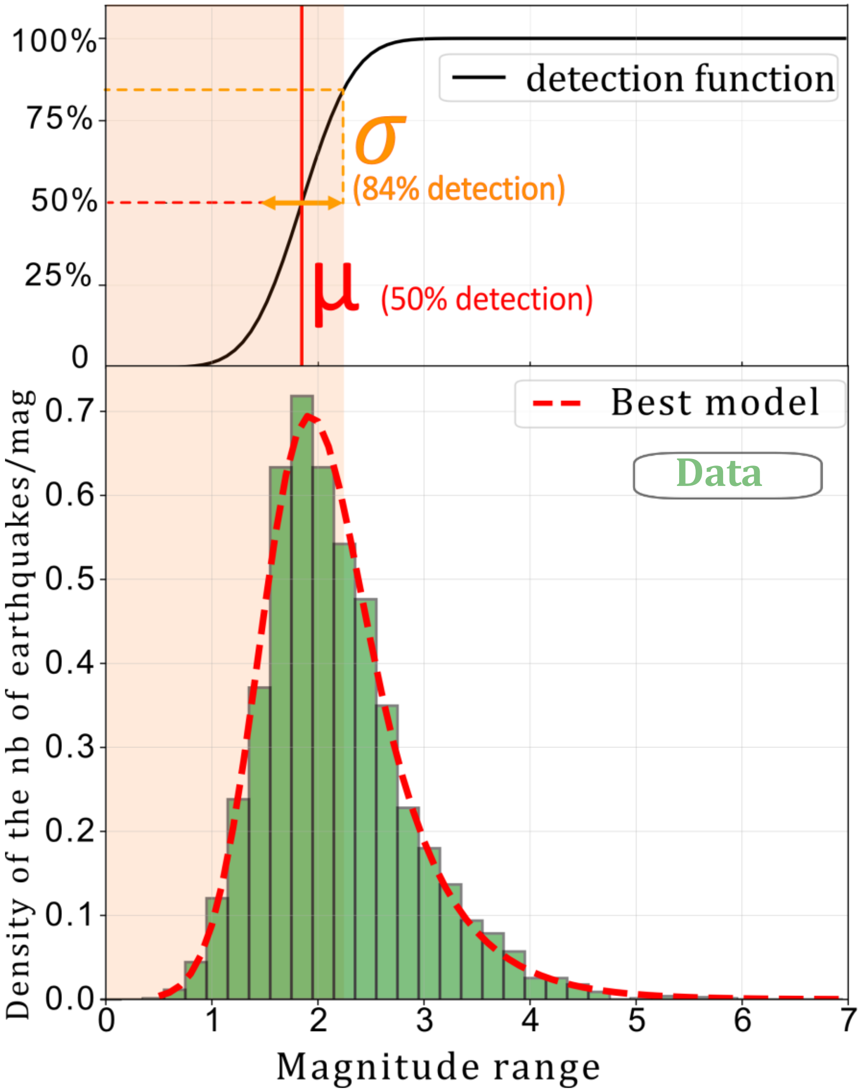

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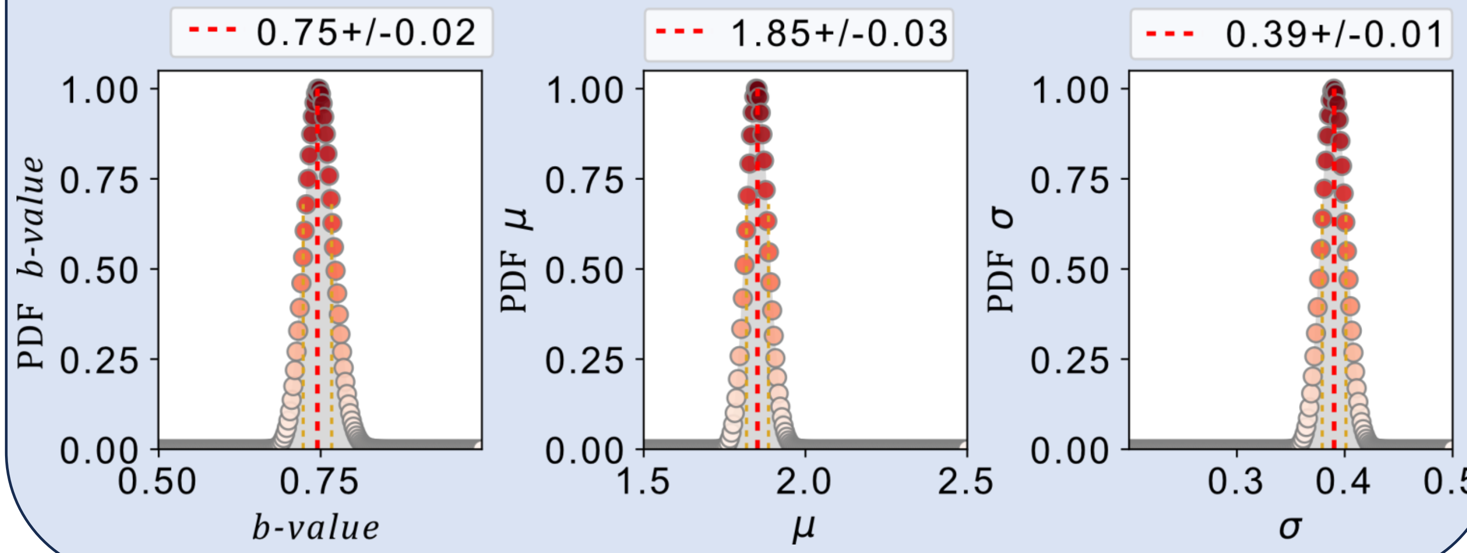
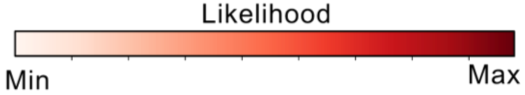
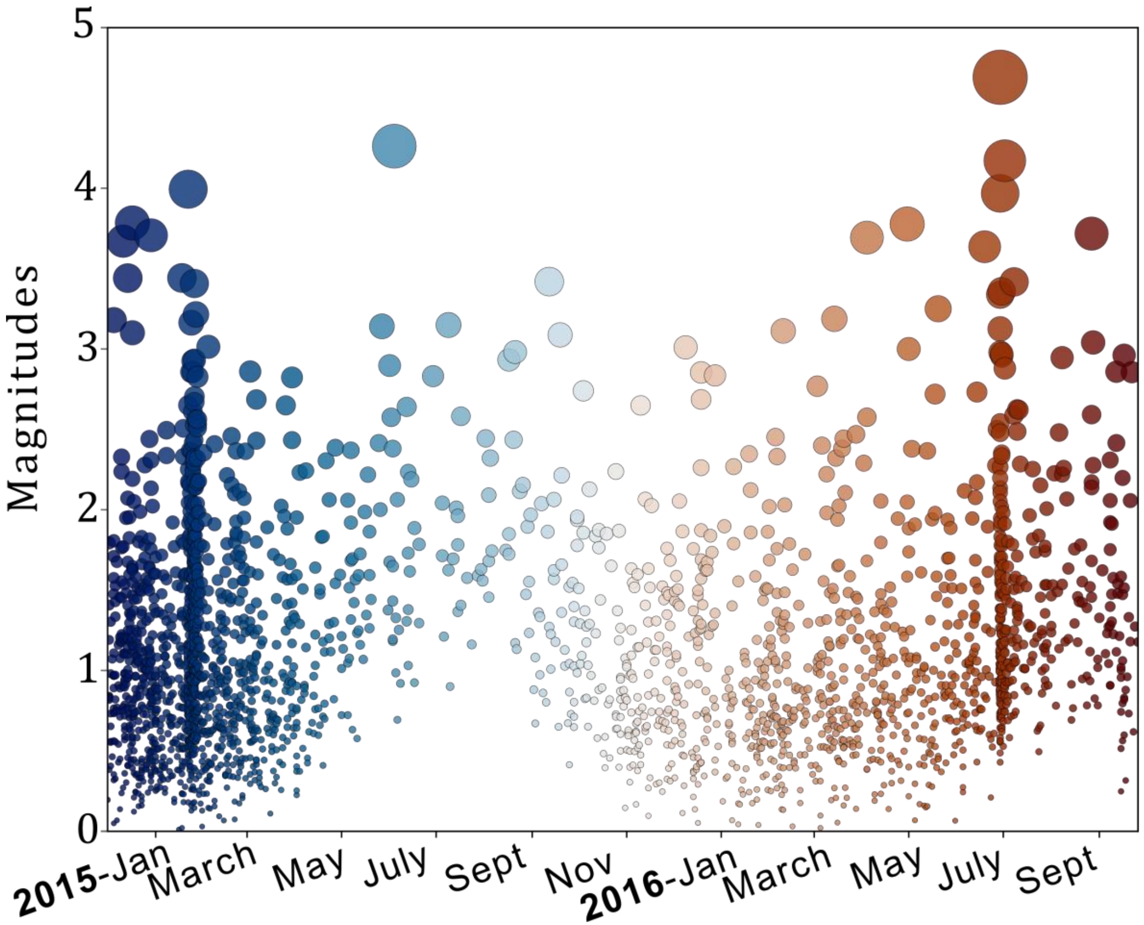




(3) Inversion of temporal changes



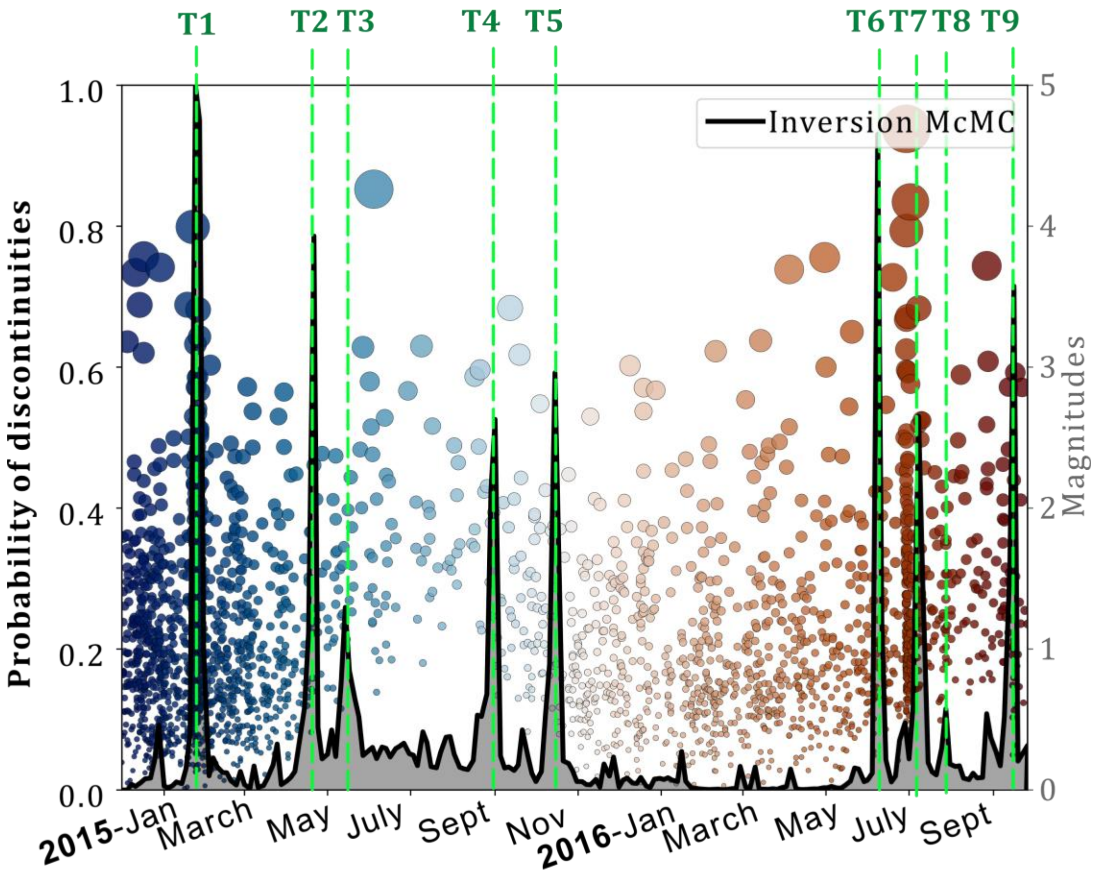
How many temporal discontinuities and **where are they ?**

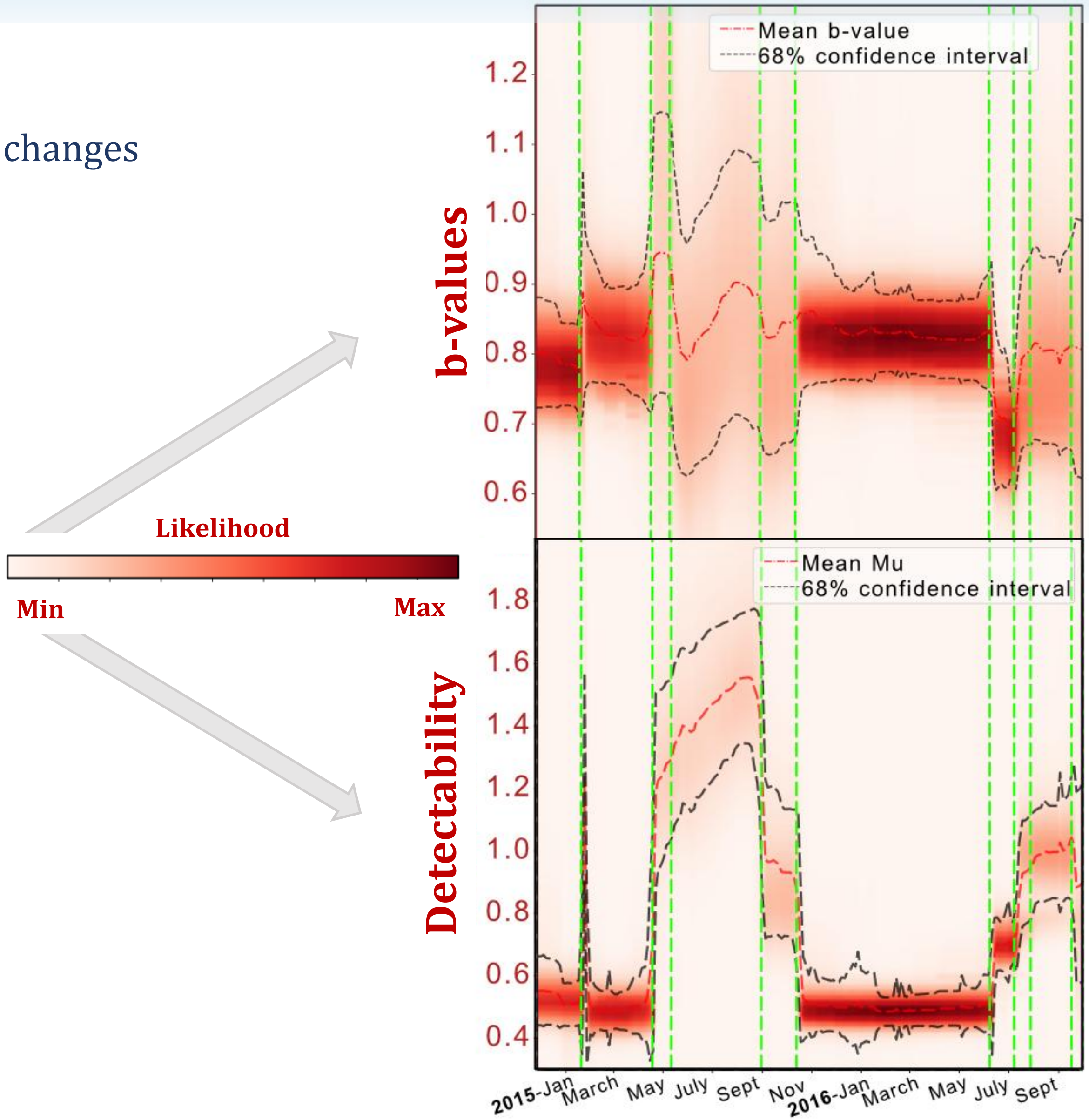
We want a method that finds automatically b_{value} changes

(3) Inversion of temporal changes

Inversion of 1D variations (space/time)

Laporte et al., 2025 : A transdimensional inversion of temporal changes using a Markov-chains Monte-Carlo framework

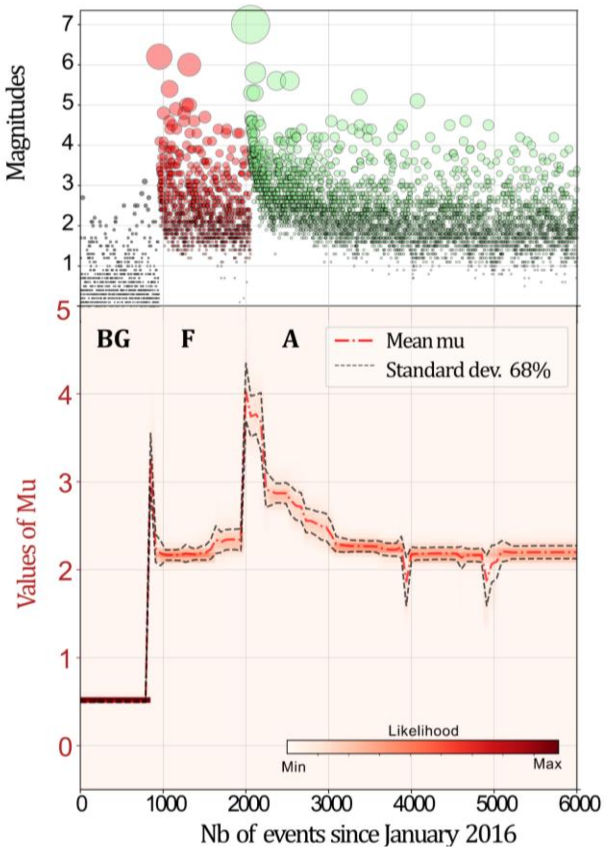


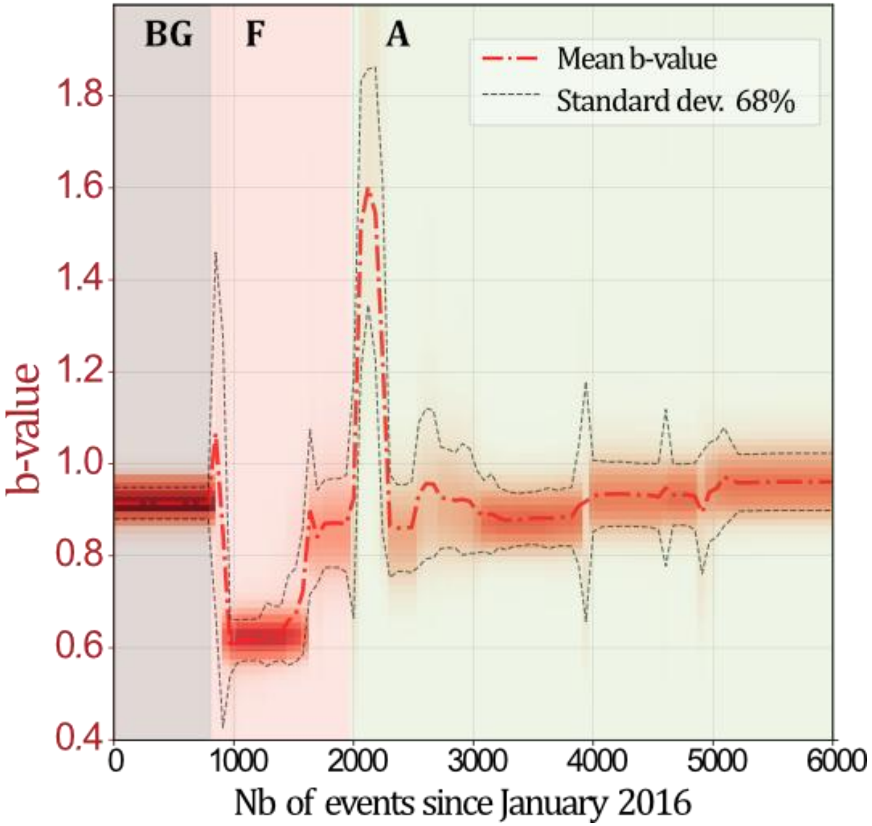


On-going project about Seismic Processes

b-GRASP

Actual project : Investigating foreshock sequences (MSCA B-GRASP)
Bayesian **G**utenberg-**R**ichter **A**nalysis for understanding **S**eismic **P**rocesses
In collaboration with : T. Bodin (ICM), S. Durand (LGL-TPE), Pierre. Arroucau (EDF)





Results of b-value temporal variations for the Kumamoto earthquake sequence
Laporte et al., in writting

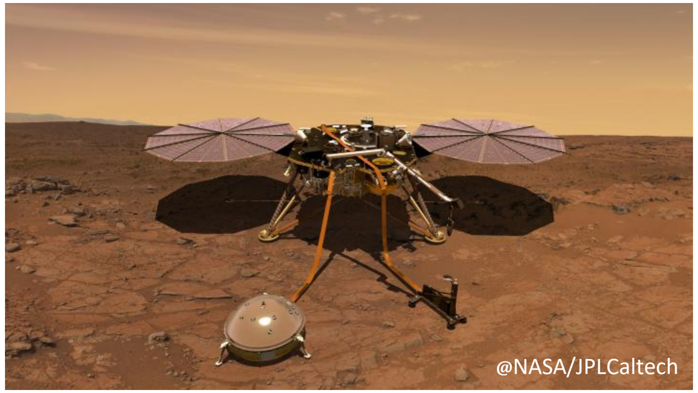
Another on-going project about b-value *(Team from Lyon)*

Swann Rubin's PhD (ENS, Lyon) : Marsquakes statistics
Supervision : Chloe Michaut (ENS), Stéphanie Durand(Univ Lyon1), Thomas Bodin (ICM)

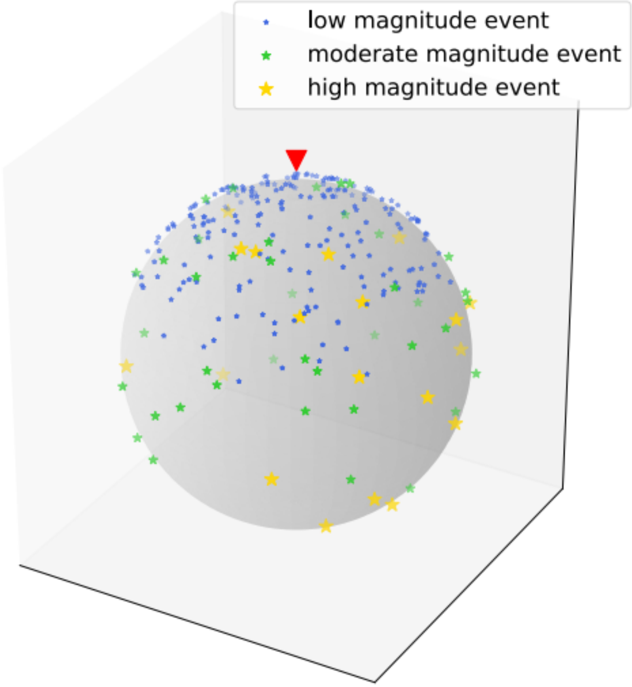
Funded by the European Union

European Research Council
Established by the European Commission

Very few Marsquakes detected :
what can we still learn about the planet's interior ?

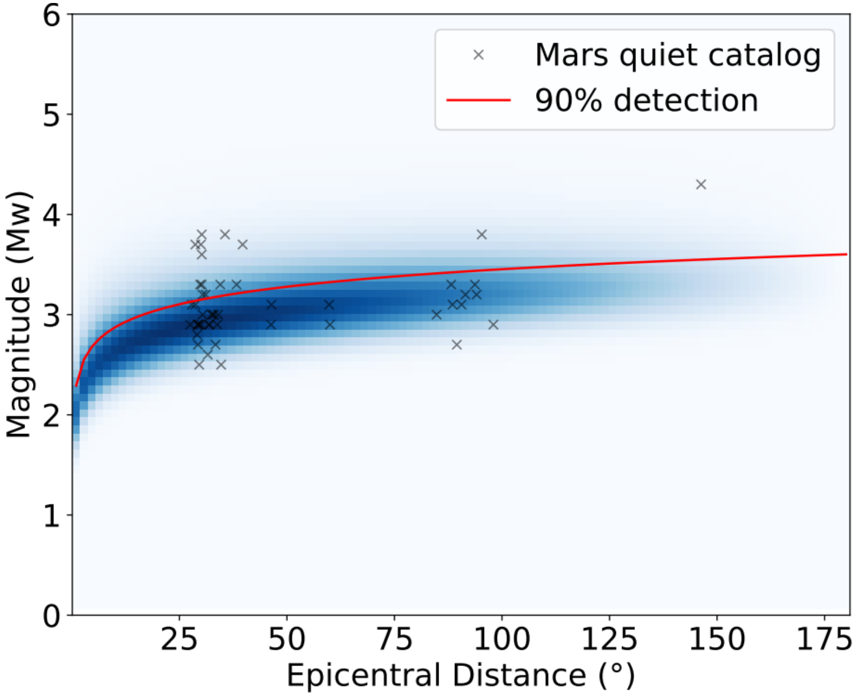


One seismic station - InSight



Model of theoretical detectability on Mars

Swann Rubin, personal communication



Swann's Results of Detectability
as a fonction of Source-Station distance

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme (grant agreement No. 101001689)

Applications for Seismic Hazard

Part 3 : Application for Seismic Hazard (C. Colin's PhD)

- (1) Adding Seismic rate as a model parameter
- (2) Adding data uncertainties
- (3) Inversion of time-varying detectability
- (4) Inversion of the space changes of b-value and seismic rate

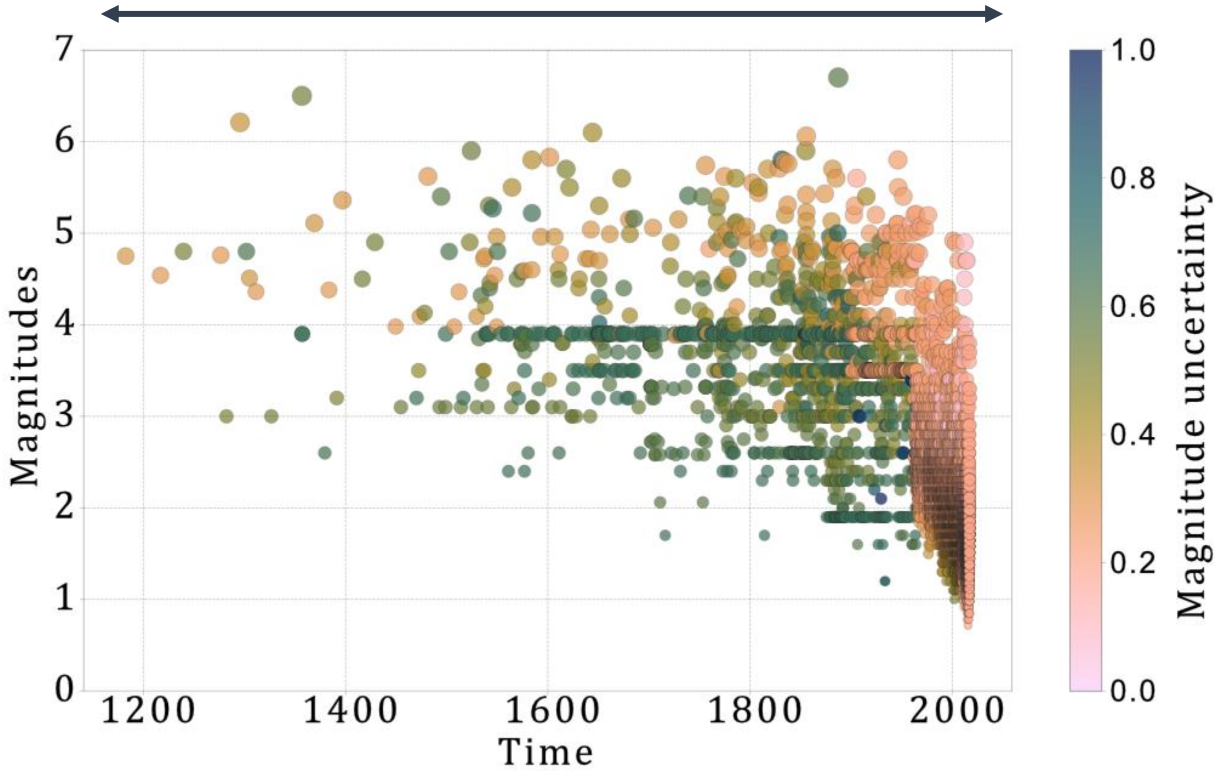
Cyrielle Colin's PhD (Univ Lyon 1, Sigma3) :

Bayesian estimation of b and a parameters for seismic hazard analysis

Supervision : Stéphanie Durand(Univ Lyon1), Thomas Bodin (ICM), Pierre Arroucau (EDF), Guillaume Daniel (EDF)

Adaptation for seismic hazard studies

b-value and Seismic rate



For each seismotectonic zones :

One long term b-value and one long-term seismic rate
(probability density functions)

Taking into account time-varying detectability

Large uncertainties in the historical data

We want a method that finds one b-value and one seismic rate while taking into account time varying detectability

(1) Adding the seismic rate as a parameter

Likelihood
Probability of **observation**

Probability of **occurence**

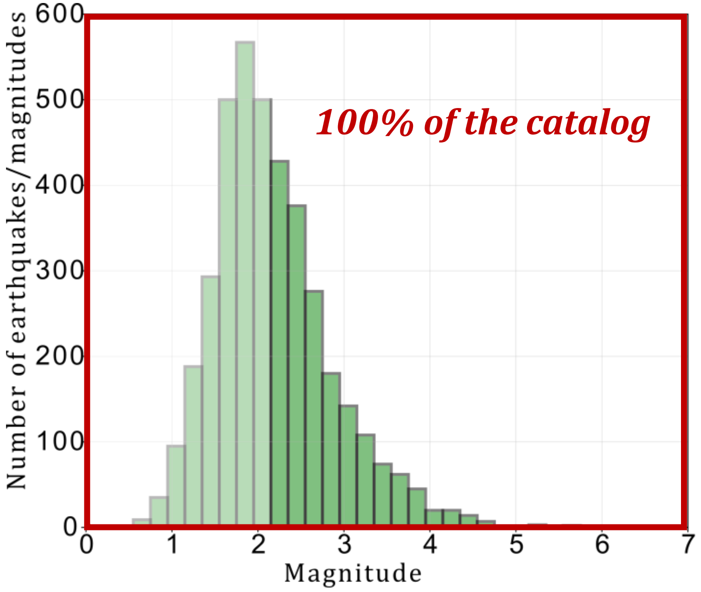
Gutenberg-Richter (1944)

Poisson process

Probability of **detection**

Ogata and Katsura (1993)

$$P(\boldsymbol{m_{obs}}, \boldsymbol{N} | \boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\sigma}, \lambda) = P(\boldsymbol{m_{obs}} | \boldsymbol{\beta}) \times P(\mathbf{N} | \lambda) \times P(Q(\boldsymbol{m_{obs}}) | \boldsymbol{\mu}, \boldsymbol{\sigma})$$



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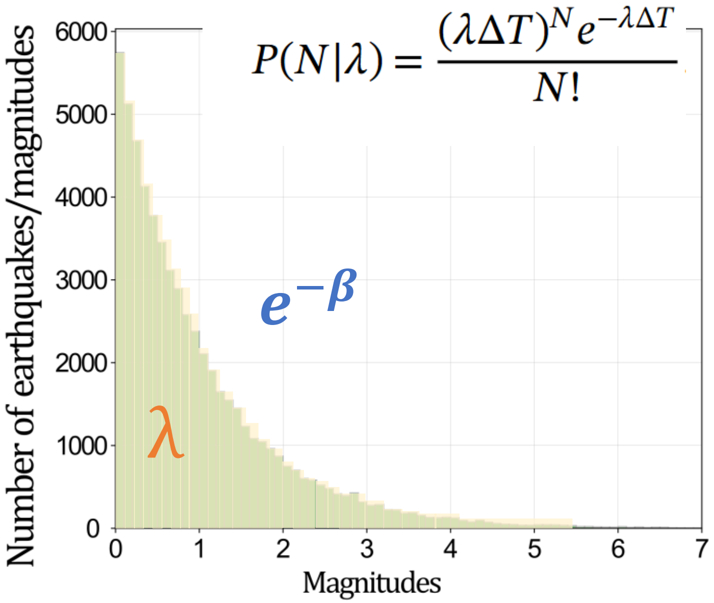

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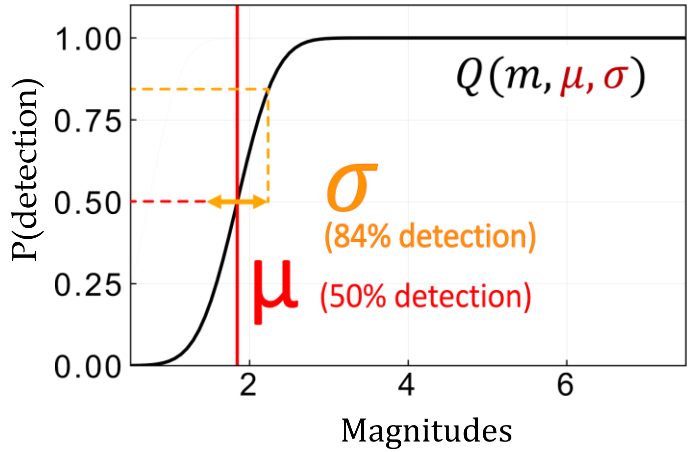


(2) Inversion of b-value and seismic rate with data uncertainties

Likelihood
Probability of **observation**

Probability of **occurence**

Gutenberg-Richter (1944)

Poisson process

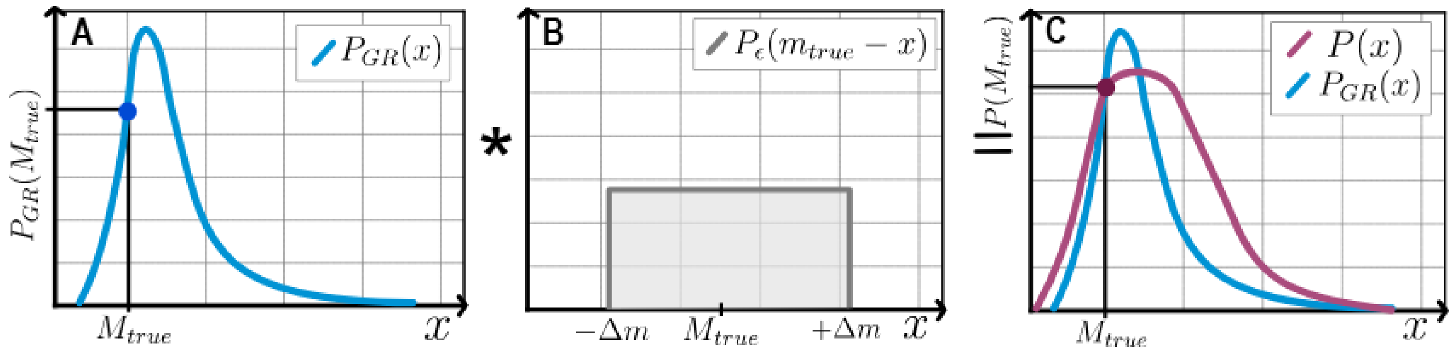
Probability of **detection**

Ogata and Katsura (1993)

$$P(\boldsymbol{m_{obs}}, \boldsymbol{N}|\boldsymbol{\beta}, \boldsymbol{\mu}, \boldsymbol{\sigma}, \lambda) = \mathrm{P}(\boldsymbol{m_{obs}}|\boldsymbol{\beta}) \times \mathrm{P}(\mathbf{N}|\lambda) \times \mathrm{P}(\mathrm{Q}(\boldsymbol{m_{obs}})|\boldsymbol{\mu}, \boldsymbol{\sigma})$$

Integration of magnitude uncertainties $\boldsymbol{m_{obs}} \pm \Delta \boldsymbol{m}$

Colin et al., under review



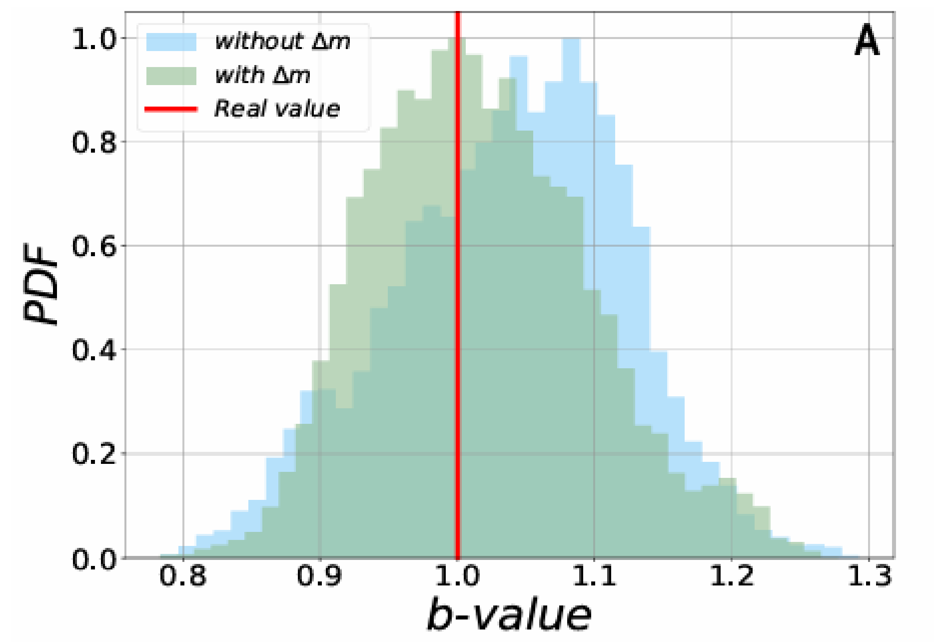
Considering a uniform distribution of magnitudes for simplification

(3) In a probabilistic framework

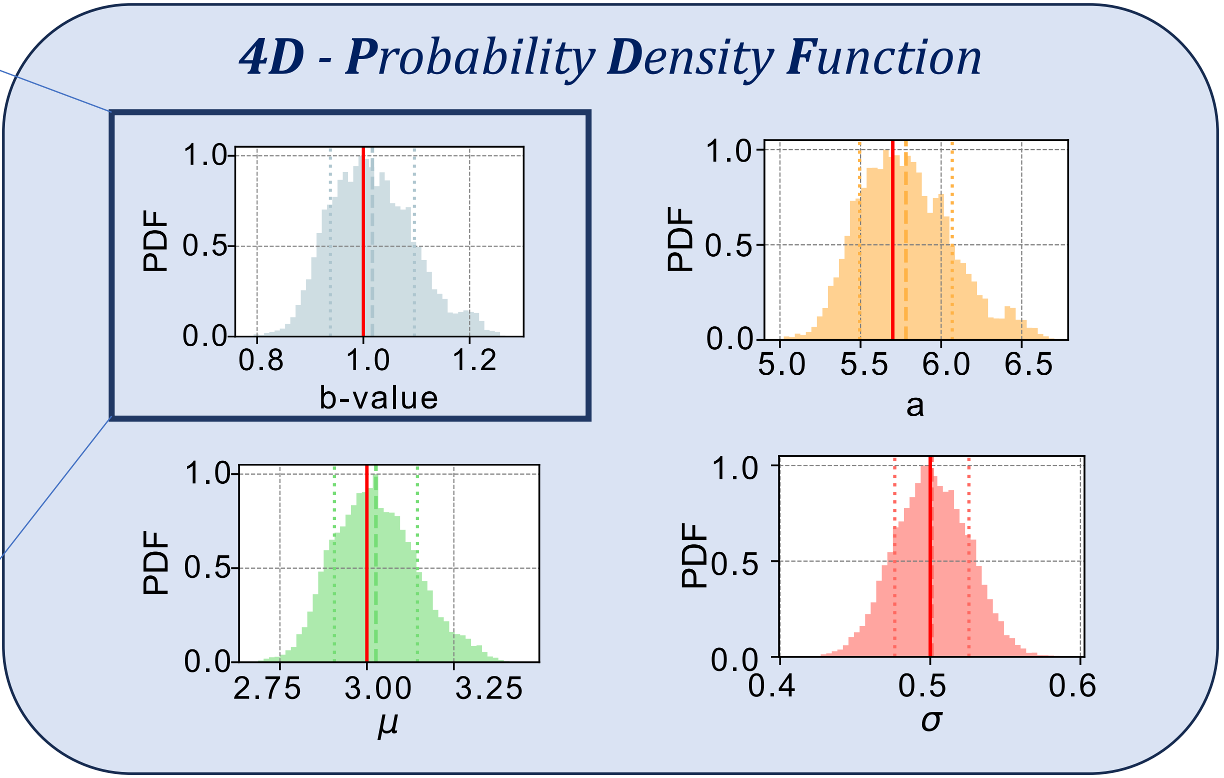
Model : (4 parameters)
{b-value , μ ,σ, lambda}

Posterior		Likelihood		Prior
$P(\text{Model}\vert m_{obs})$	$\propto$	$P(m_{obs}\vert Model)$	$\times$	$P(Model)$

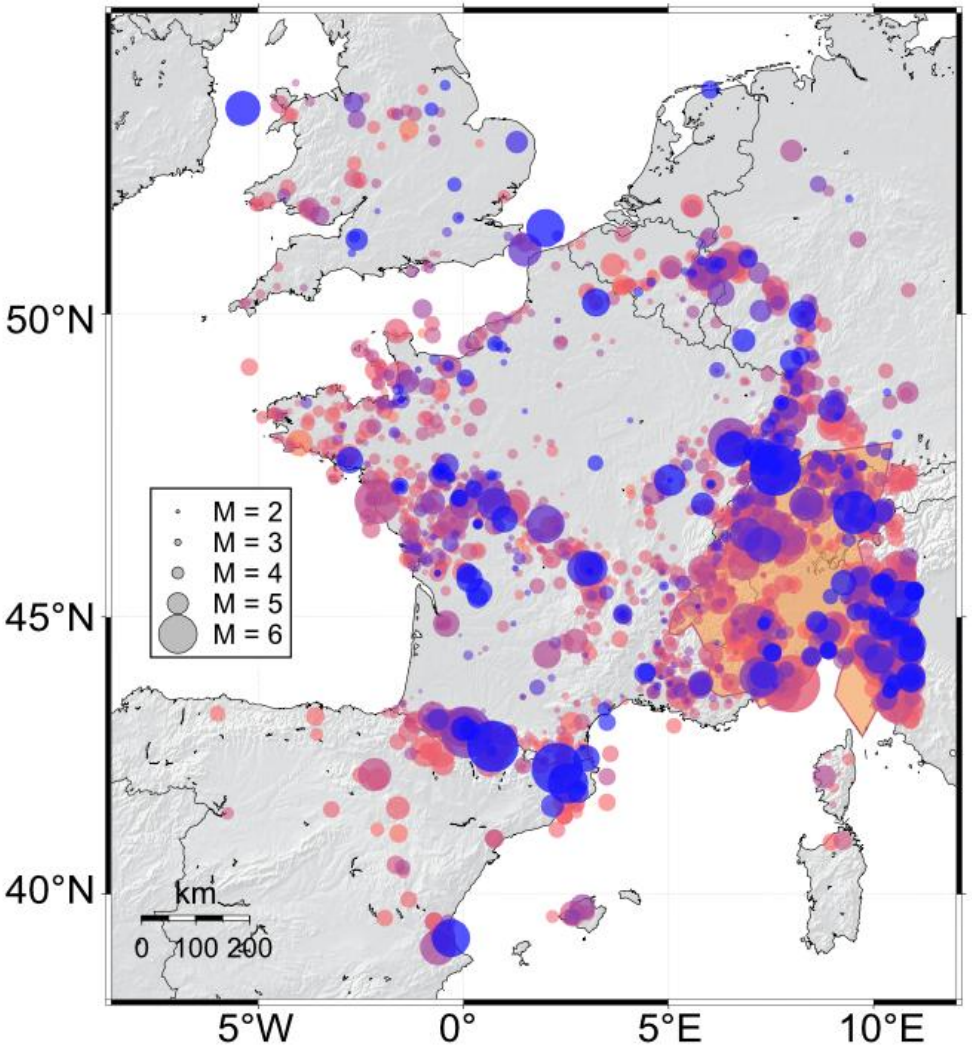
Comparing with or without uncertainties



Figures from Colin et al., in review

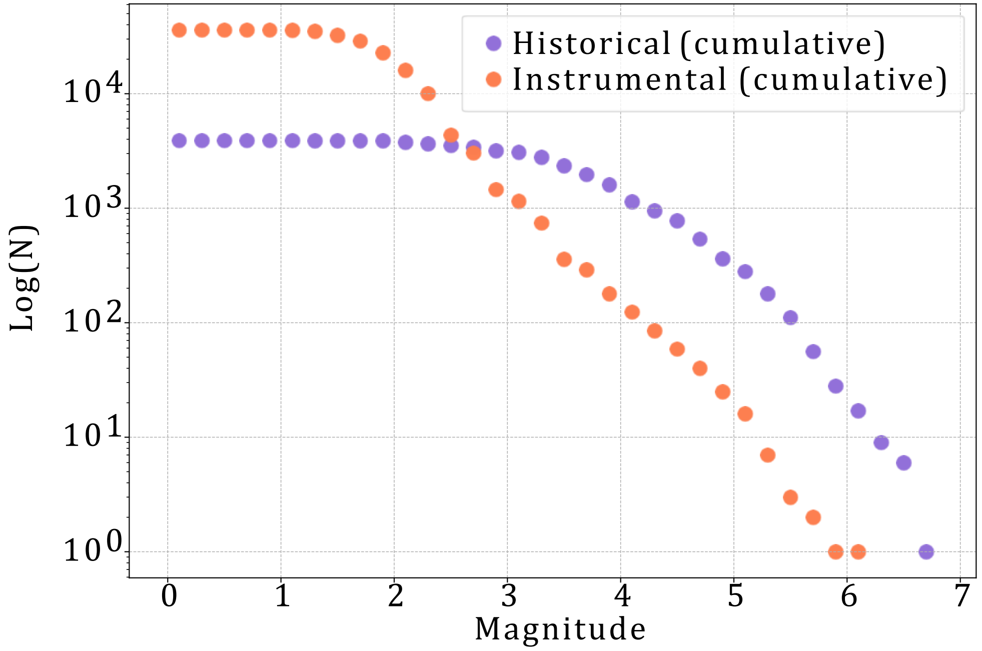


Application to French Seismicity : Alpine zone



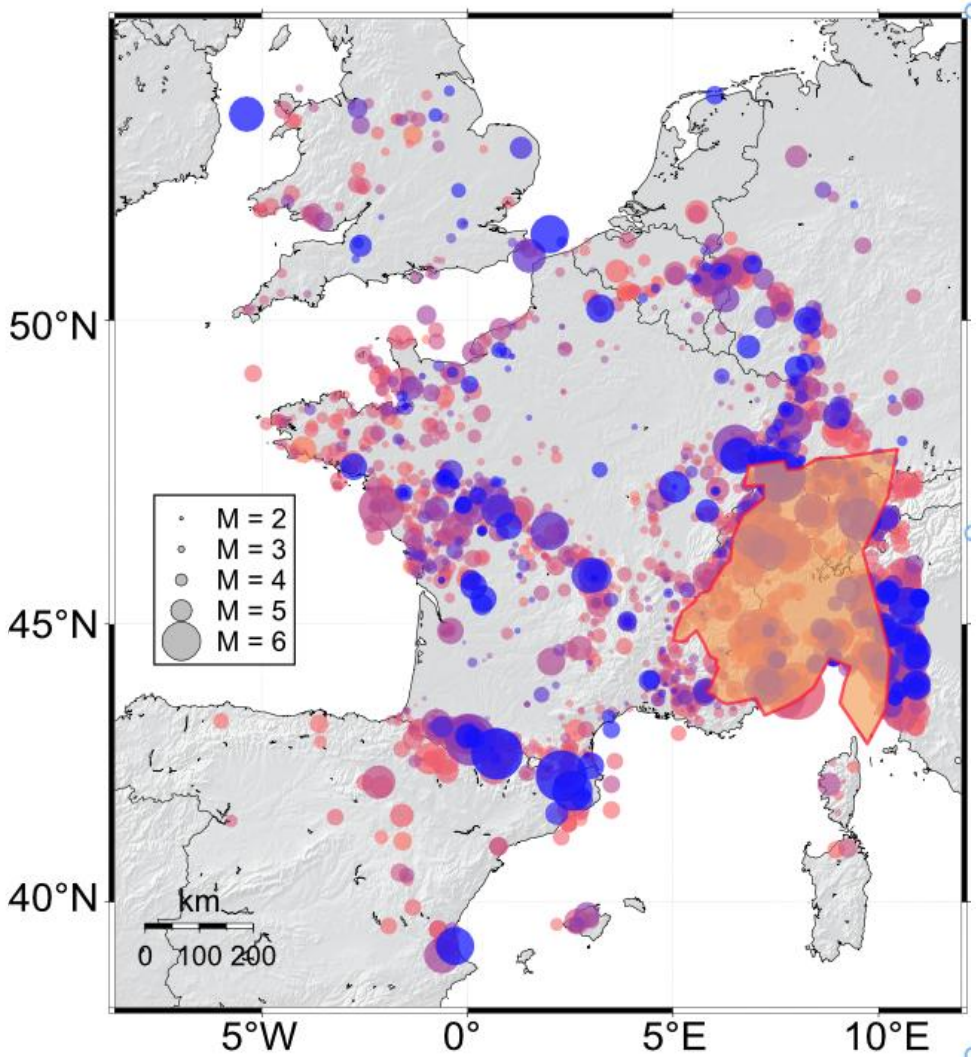
Earthquake catalog from Drouet et al., 2020

Adding information with large earthquakes occurrences from **historical** catalogs



Figures from Colin et al., in review

Application to French Seismicity : Alpine zone

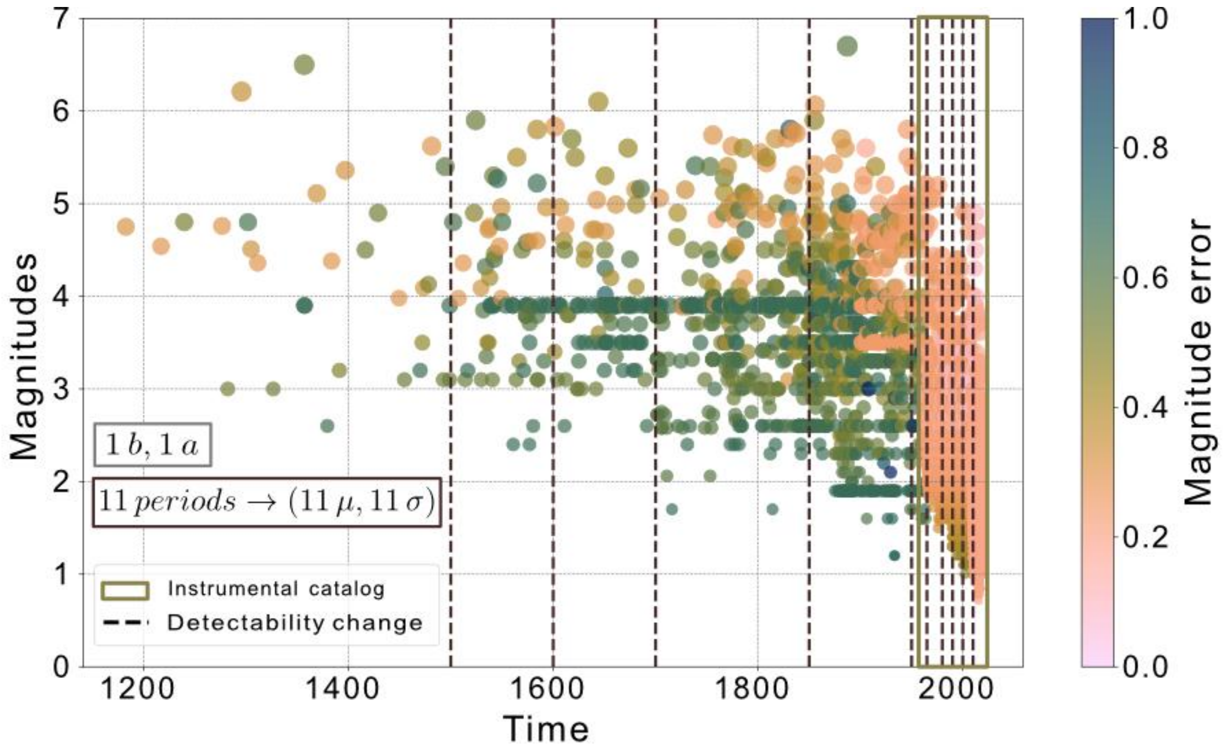


Earthquake catalog from Drouet et al., 2020

Inverting for **detectability** in each period

11 periods in Historical and Instrumental period

Colin et al., in review



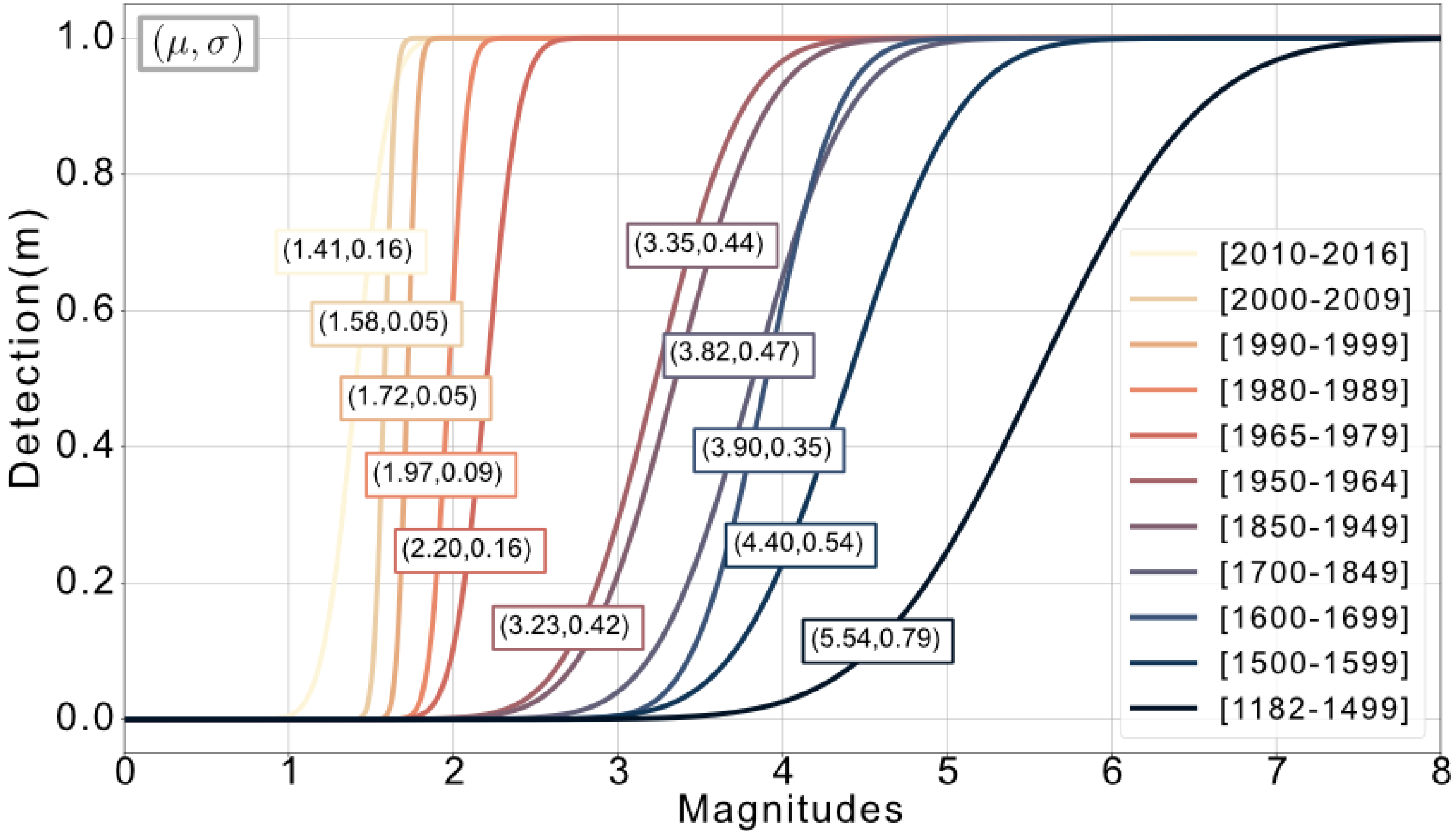
Figures from Colin et al., in review

Results for French Seismicity : Alpine zone

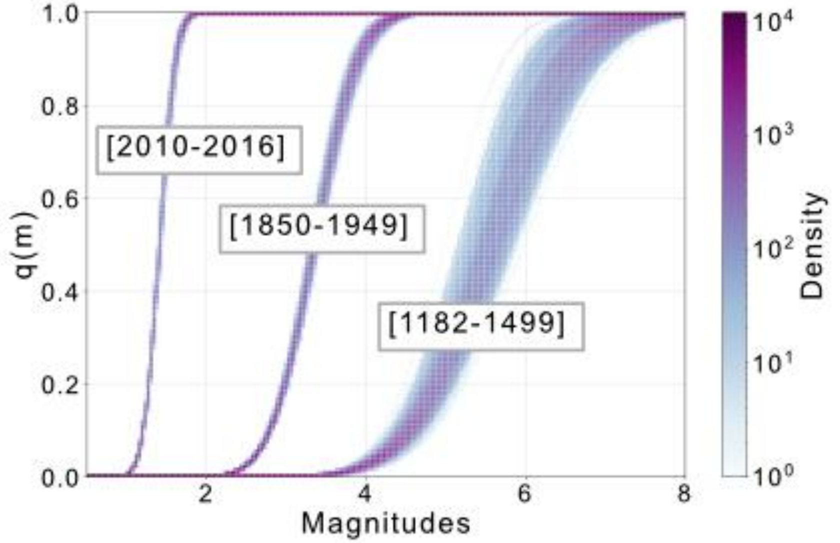
Inversion of the temporal variability of the detectability

Instrumental
(low mu, low sigma)

Historical
(high mu, high sigma)



Larger spread of the probability density functions for historical part

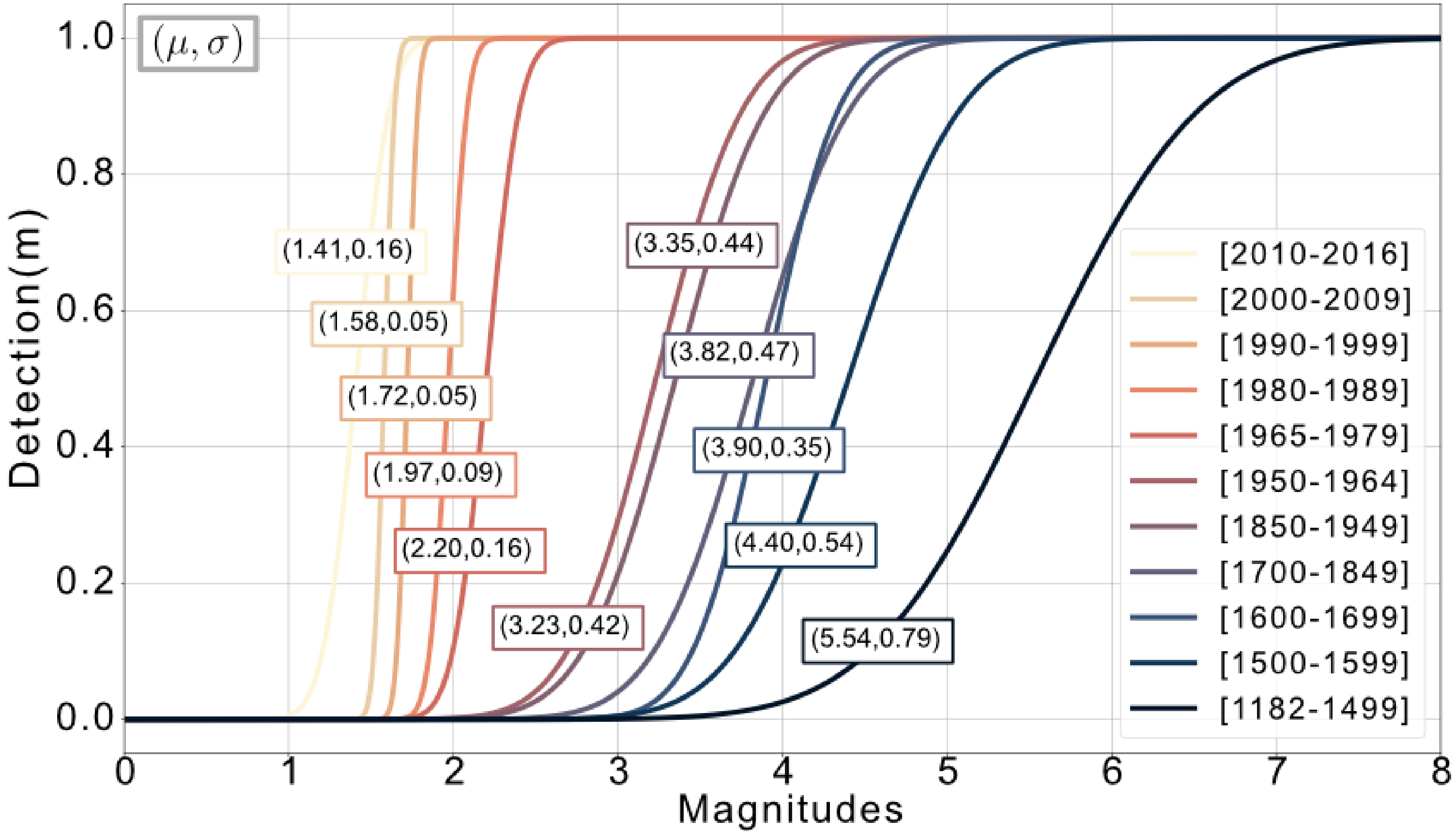


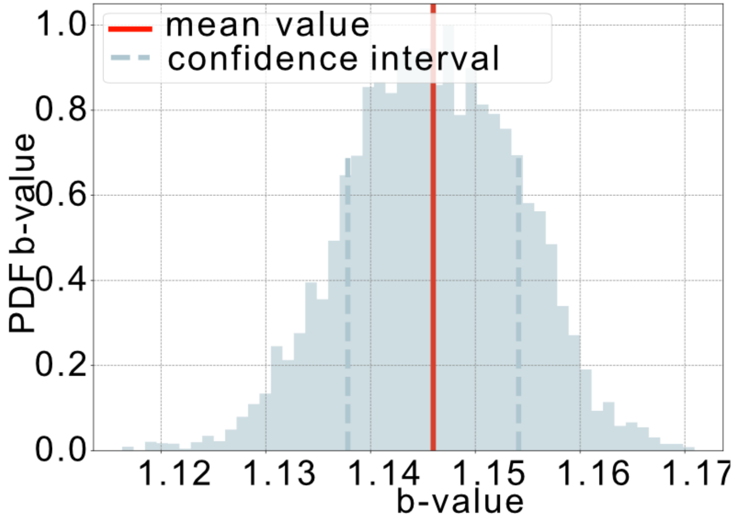
Figures from Colin et al., in review

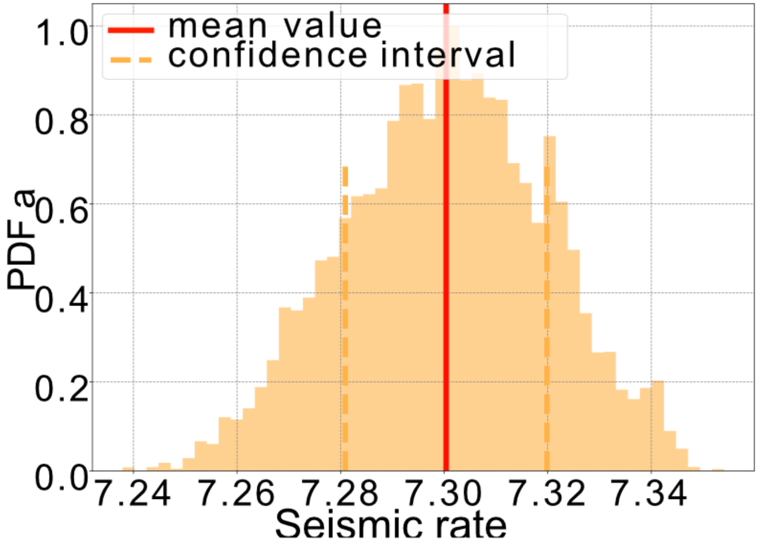
Results for French Seismicity : Alpine zone

Inversion of the temporal variability of the detectability

Assumption : the b-value and the seismic rate are constant over a long period of time

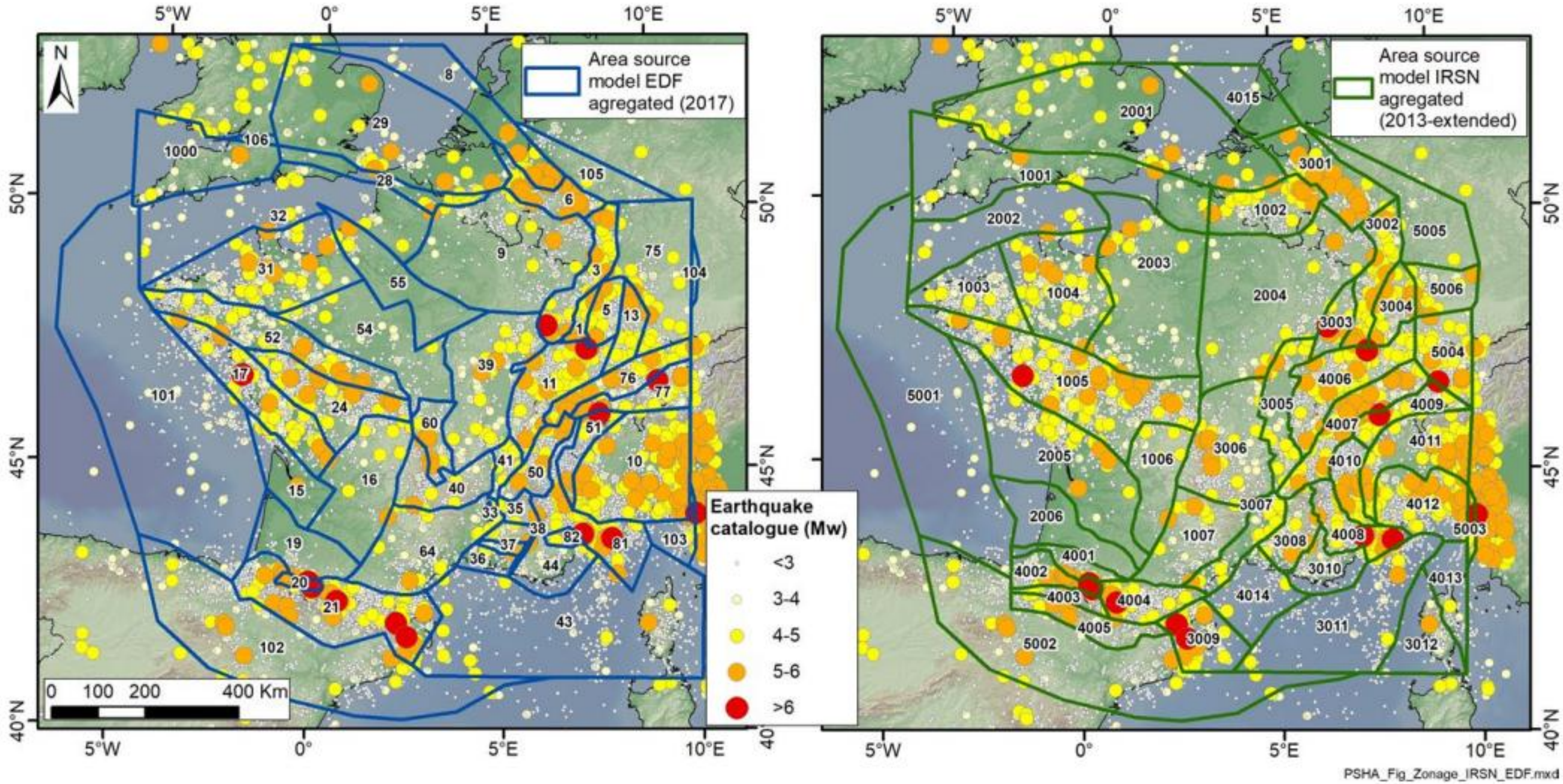






Figures from Colin et al., in review

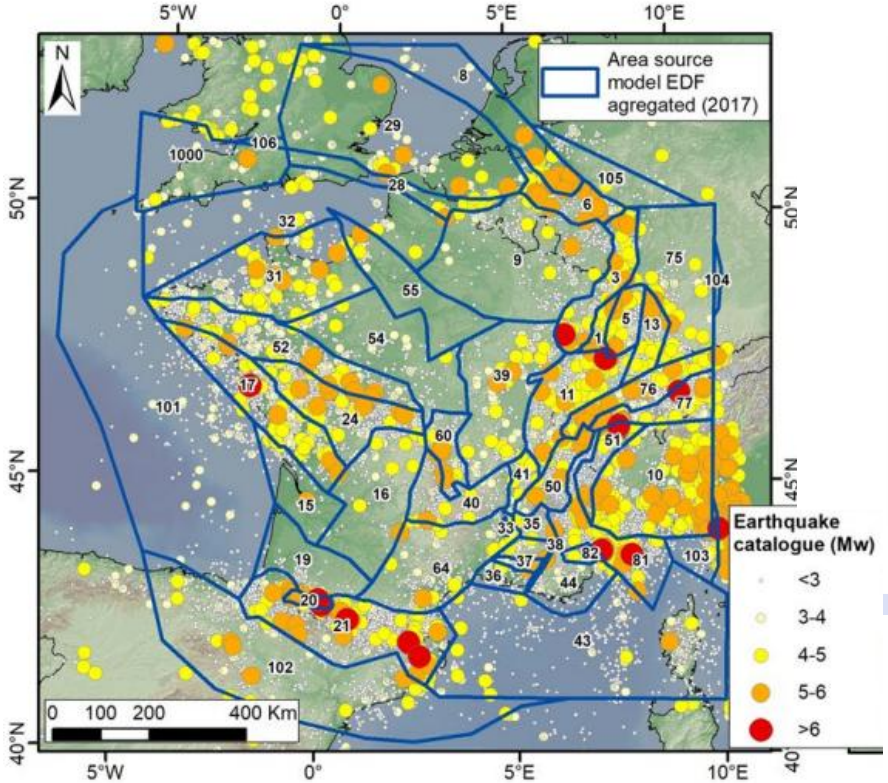
Perspectives : questioning seismotectonic zones

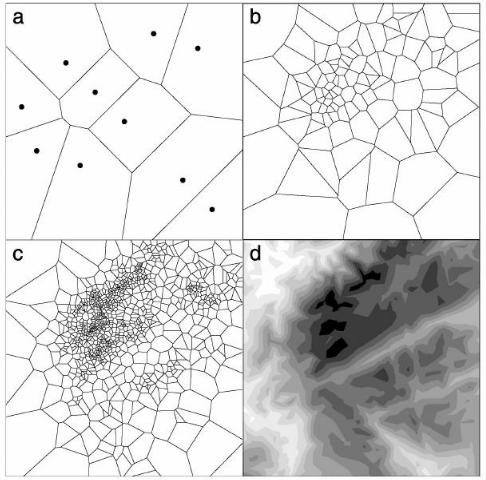


Comparison of EDF and IRSN tectonic zonations *(Drouet et al., 2020)*

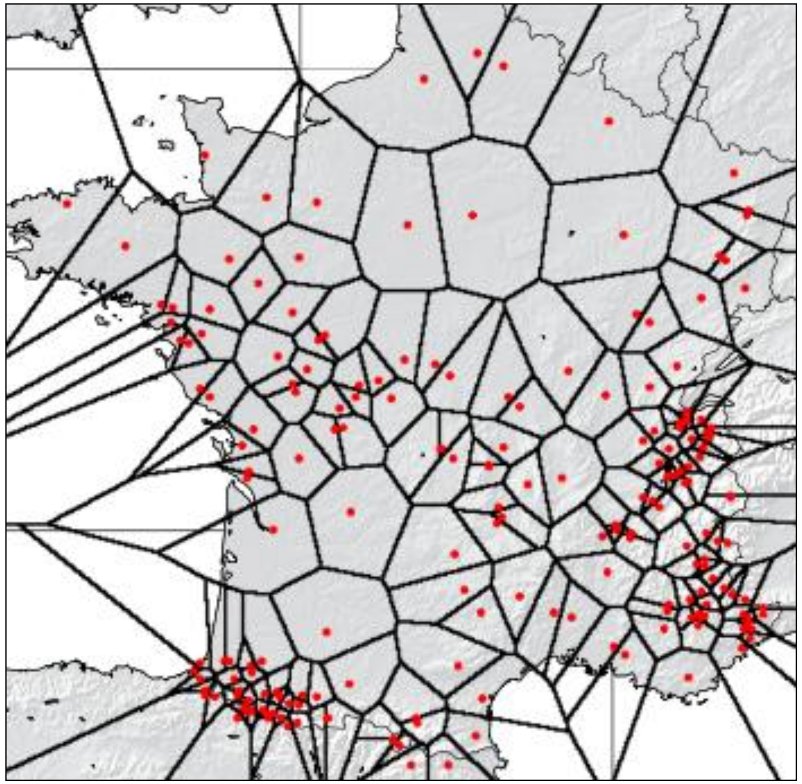
Perspectives : questioning seismotectonic zones

In 2026 : J. Renac M2 internship, M. Laporte, T. Bodin, S. Durand, C. Colin
2D Transdimensional inversion of b and a-values spatial distributions





Inversion of
kernels of voronoï cells
Probabilistic framework

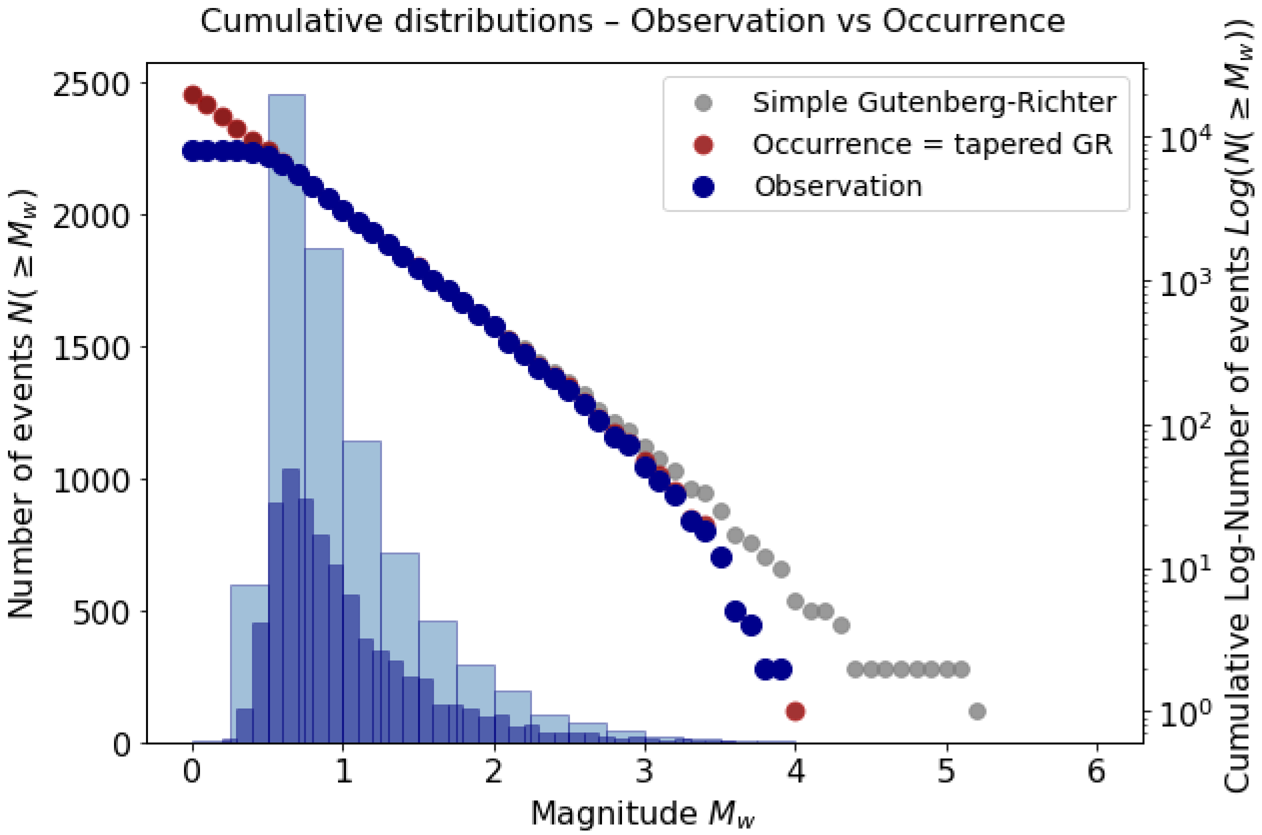


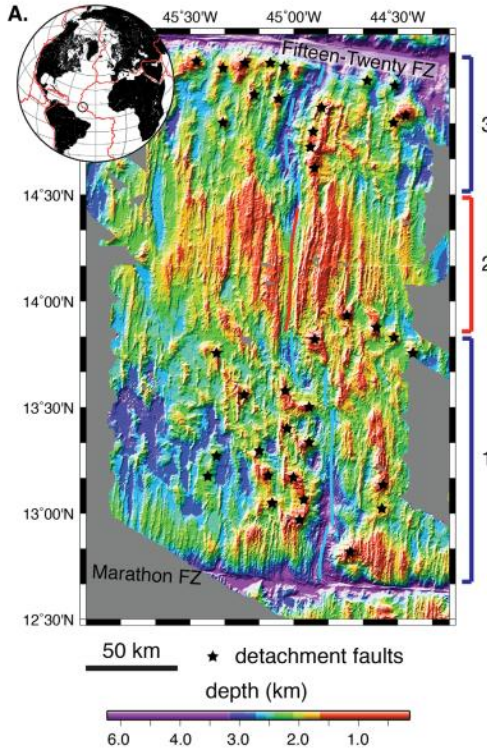
Voronoi cells sketch for French seismicity
Courtesy of Theo Santos (ICM)

Perspectives : tapered Gutenberg-Richter

On-going : Adding a taper for the maximum magnitude of earthquake catalog

In Collaboration with Thomas Bodin (ICM), Stéphanie Durand (Lyon1), Jean Arthur Olive (ENS, Paris), Pierre Yves Raumer (ENS, Paris)





In order to estimate the total moment (energy) released from earthquakes for oceanic ridges (Olive and Escartin., 2016)

Conclusions and Perspectives
A constellation of studies

ICM Barcelone Team

Thomas Bodin

Julien Renac (M2)

Céline Leblanc (PhD)

ISTerre – Chambéry team

Blandine Gardonio

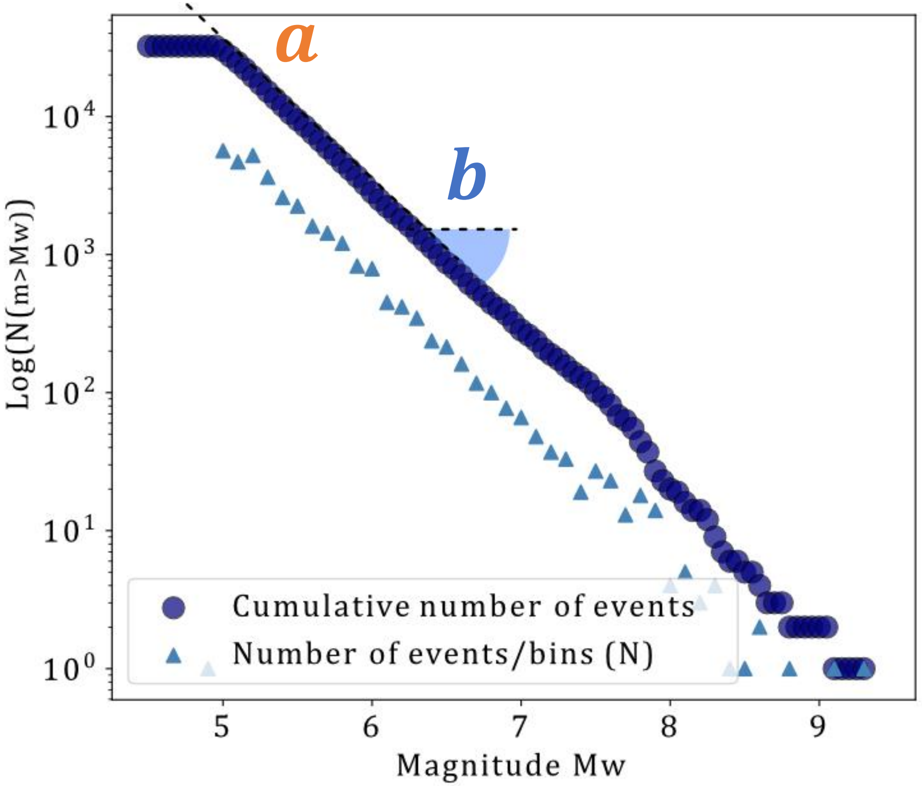
David Marsan

LGL-TPE Lyon team

Cyrielle Colin

Stéphanie Durand

Earthquakes statistics for understanding **seismic processes**



Earthquakes statistics for **seismic hazard assessment applications**

Marsquakes team (ENS, Lyon)

Chloe Michaut

Swann Rubin

EDF team

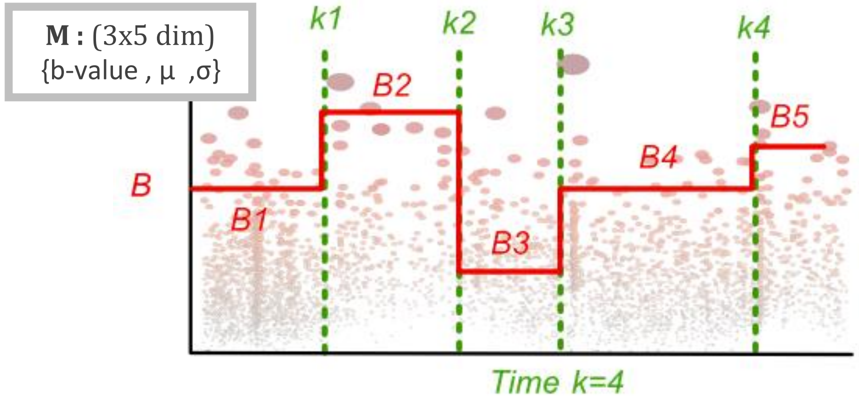
Pierre Arroucau

Guillaume Daniel

(3) Inversion of temporal discontinuities

Transdimensional Markhov-chain Monte-Carlo (McMC) *(Green, 1995; Bodin and Sambridge., 2009)*:

Extension of McMC to spaces of **varying dimension**
-> An **iterative stochastic approach** to explore the space of **models** (~ number and position of temporal discontinuities)

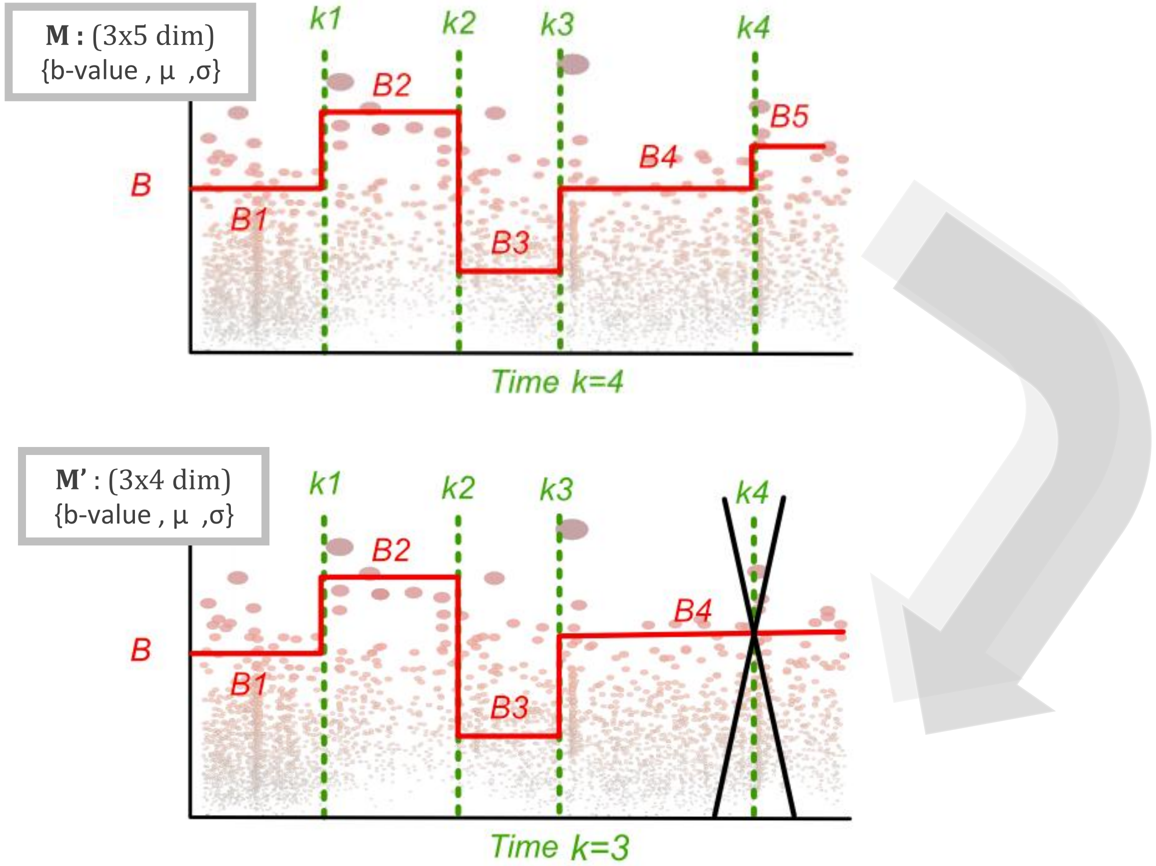


We want a method that finds automatically b_{value} changes

(3) Inversion of temporal discontinuities

Transdimensional Markhov-chain Monte-Carlo (McMC) *(Green, 1995; Bodin and Sambridge., 2009)*:

Extension of McMC to spaces of **varying dimension**
-> An **iterative stochastic approach** to explore the space of **models** (~ number and position of temporal discontinuities)



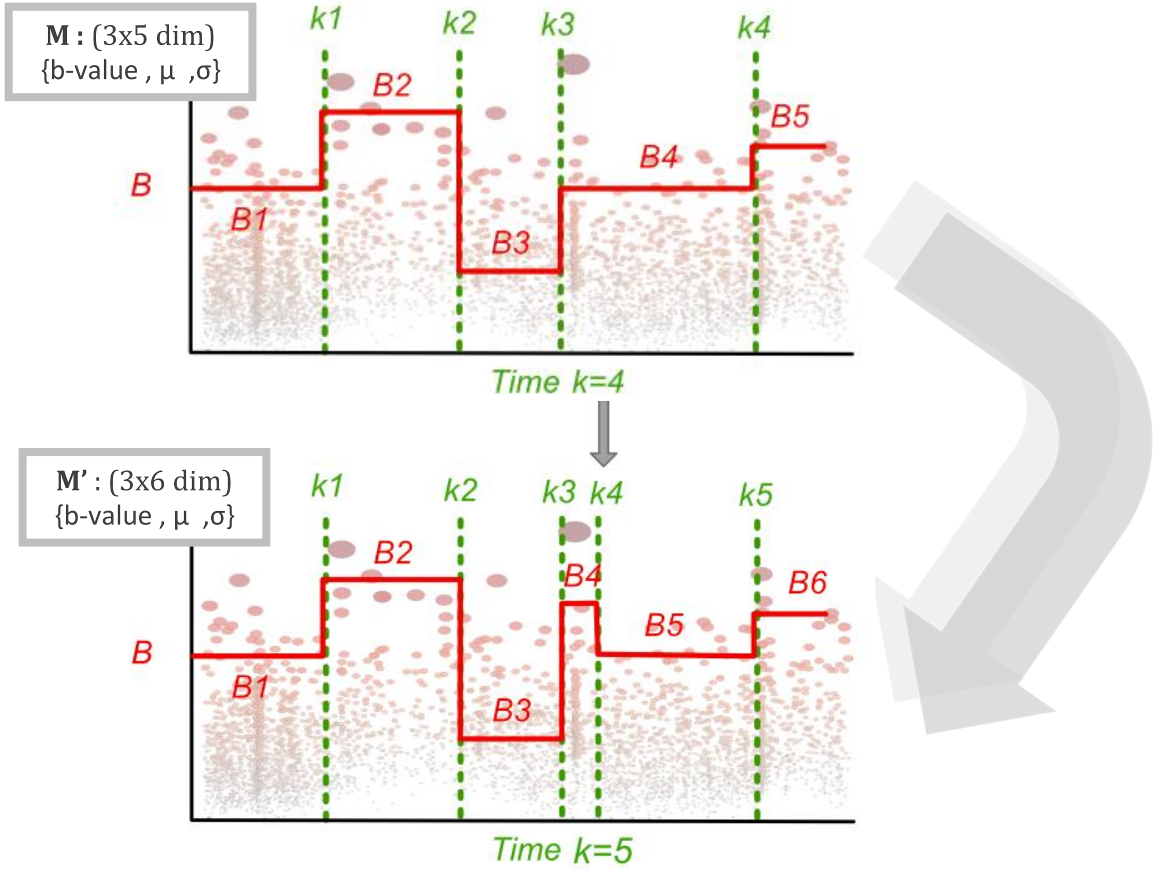
- **Updated model : M'_{update}**
  - Death of a discontinuity (k=3)

  random choice

(3) Inversion of temporal discontinuities

Transdimensional Markhov-chain Monte-Carlo (McMC) *(Green, 1995; Bodin and Sambridge., 2009)*:

Extension of McMC to spaces of **varying dimension**
-> An **iterative stochastic approach** to explore the space of **models** (~ number and position of temporal discontinuities)



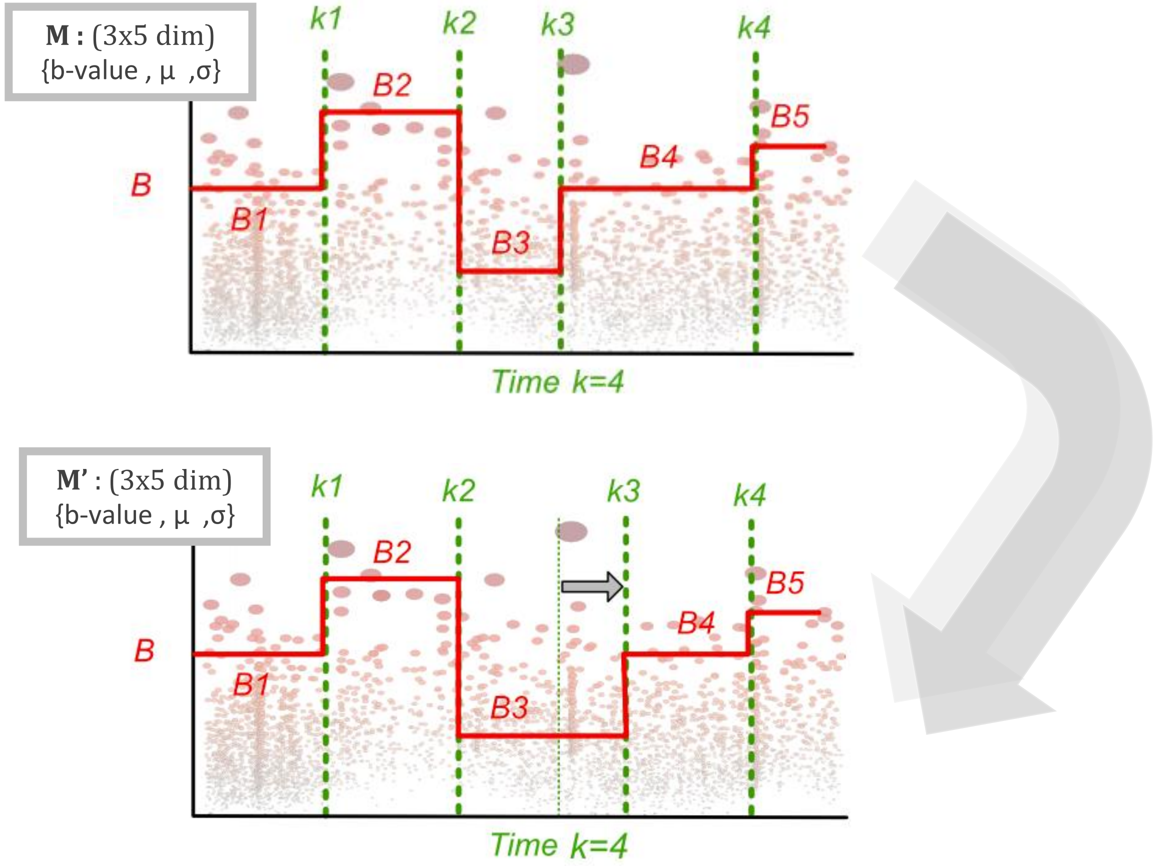
- **Updated model : M'_{update}**
  - Death of a discontinuity (k=3)
  - Birth of a discontinuity (k=5)

  random place

(3) Inversion of temporal discontinuities

Transdimensional Markhov-chain Monte-Carlo (McMC) *(Green, 1995; Bodin and Sambridge., 2009)*:

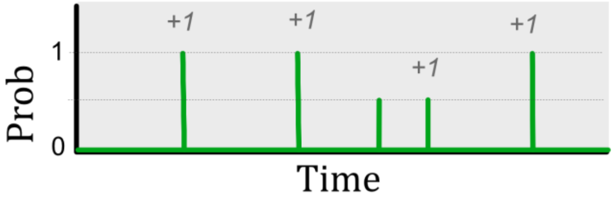
Extension of McMC to spaces of **varying dimension**
-> An **iterative stochastic approach** to explore the space of **models** (~ number and position of temporal discontinuities)



- **Updated model : M'_{update}**
  - Death of a discontinuity (k=3)
  - Birth of a discontinuity (k=5)
  - **Move** of a discontinuity (k=4)

$$k3 \mp \sigma_{move}$$

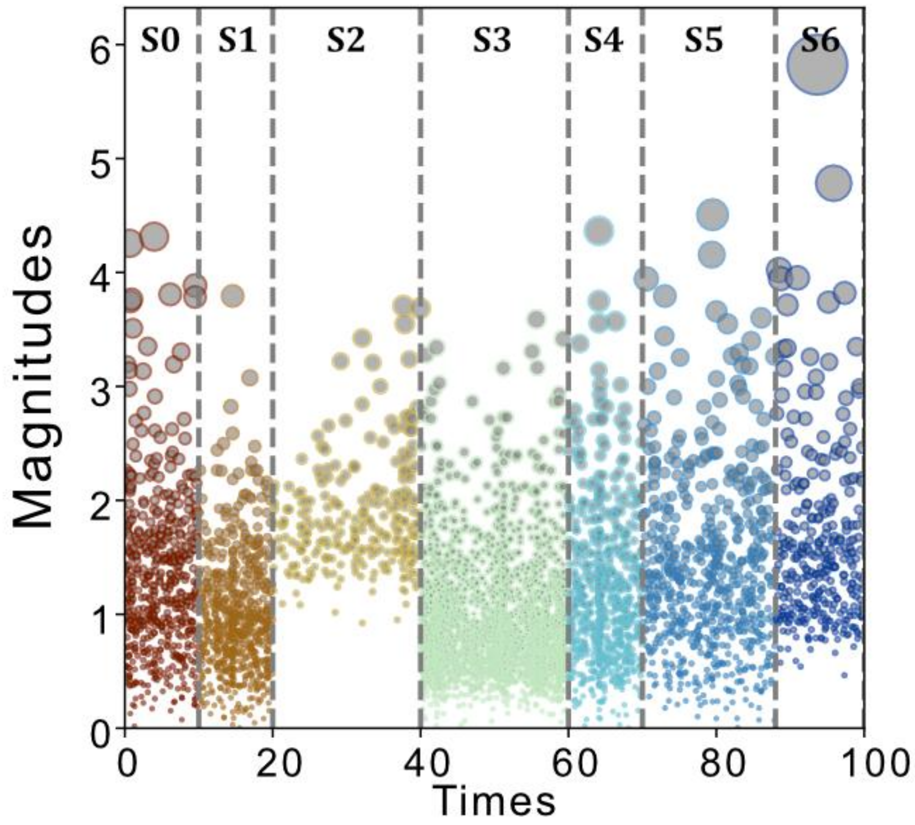
If accepted *(McMC acceptance $\boldsymbol{\alpha}$)*
M→ M'



(3) Inversion of temporal changes

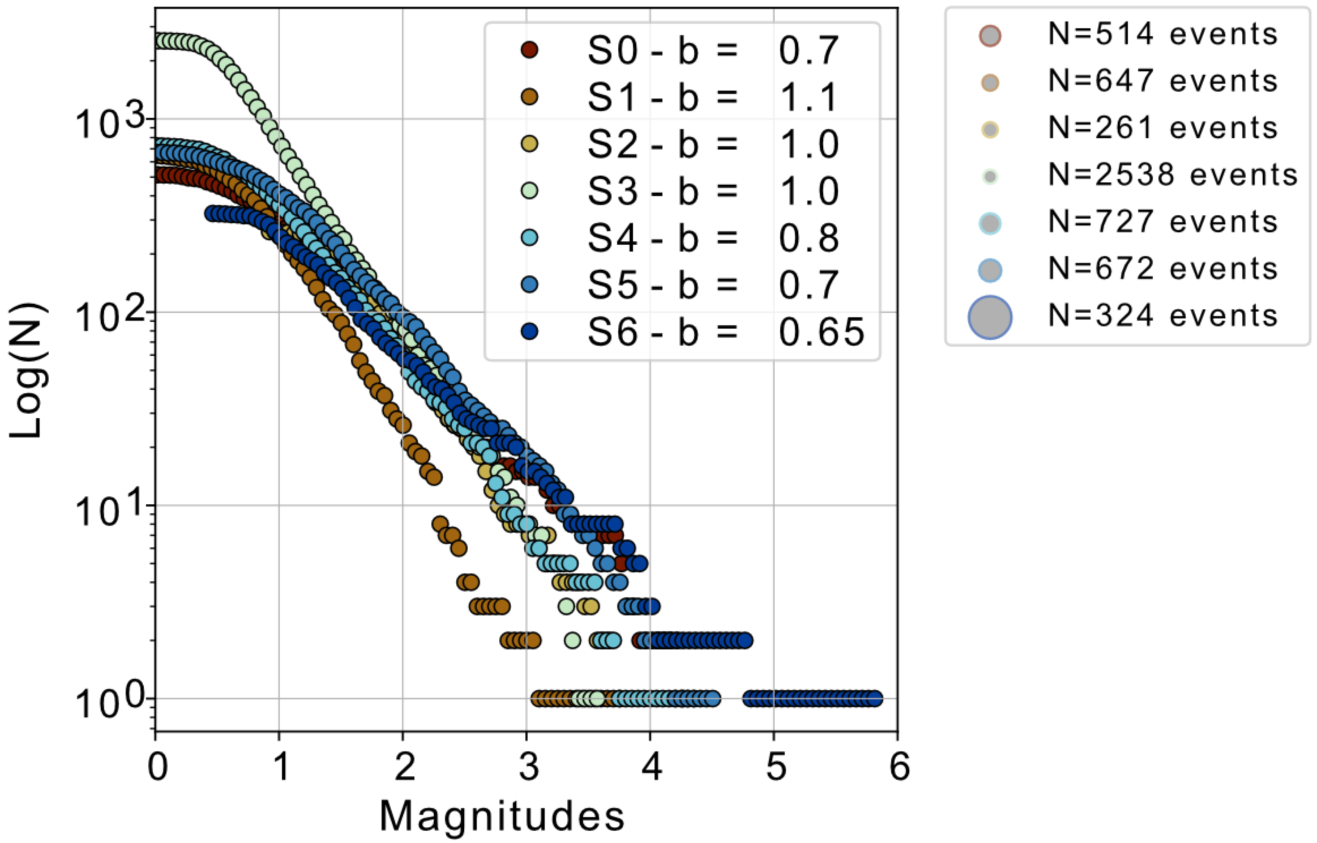
Synthetics : Gutenberg-Richter (b) - Detection law (μ,σ) - Etas

- 5600 events / 6 discontinuities
- Generated temporal variations of b, μ, σ

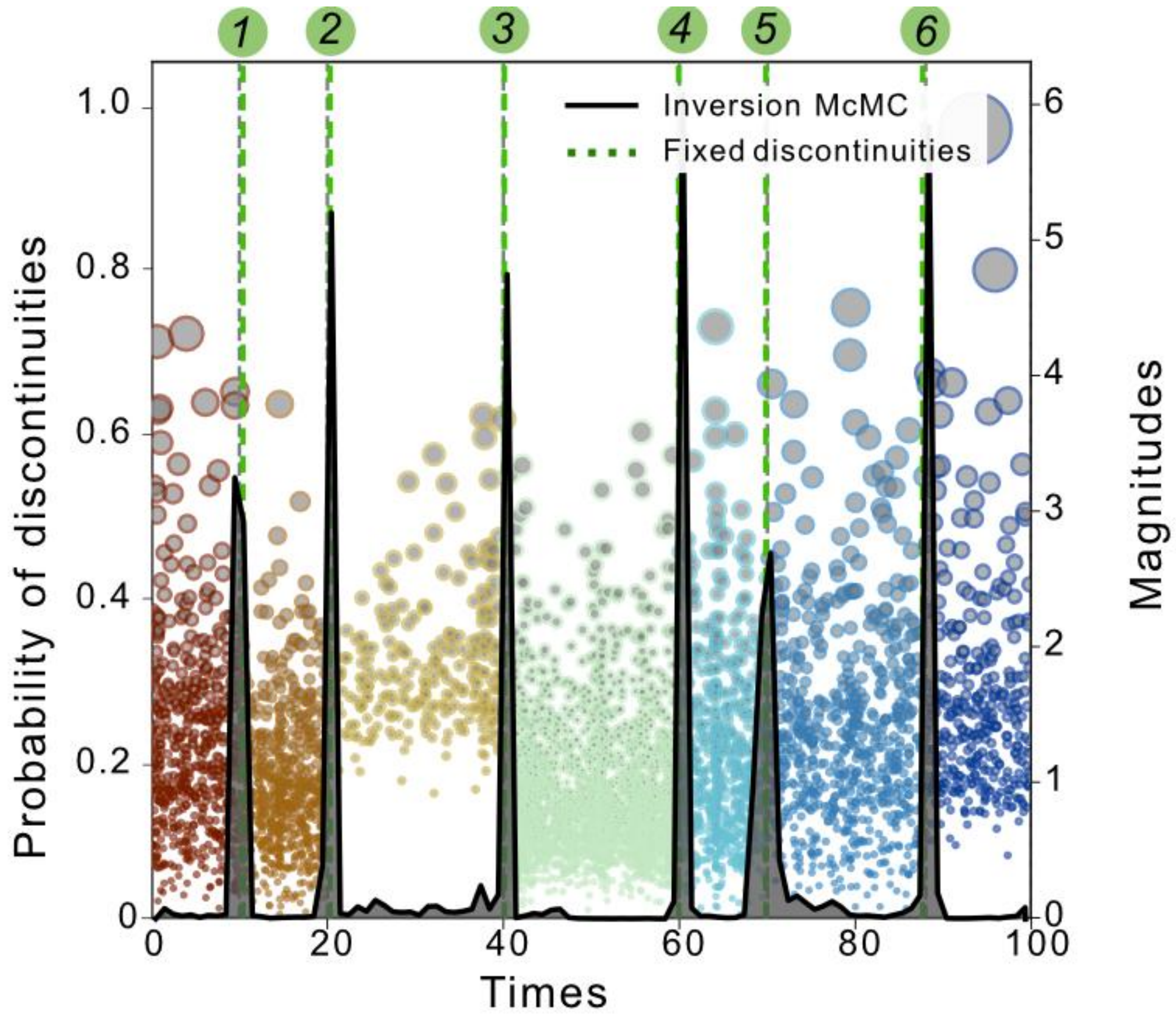


Frequency-Magnitude distribution

Synthetics – 6 discontinuities

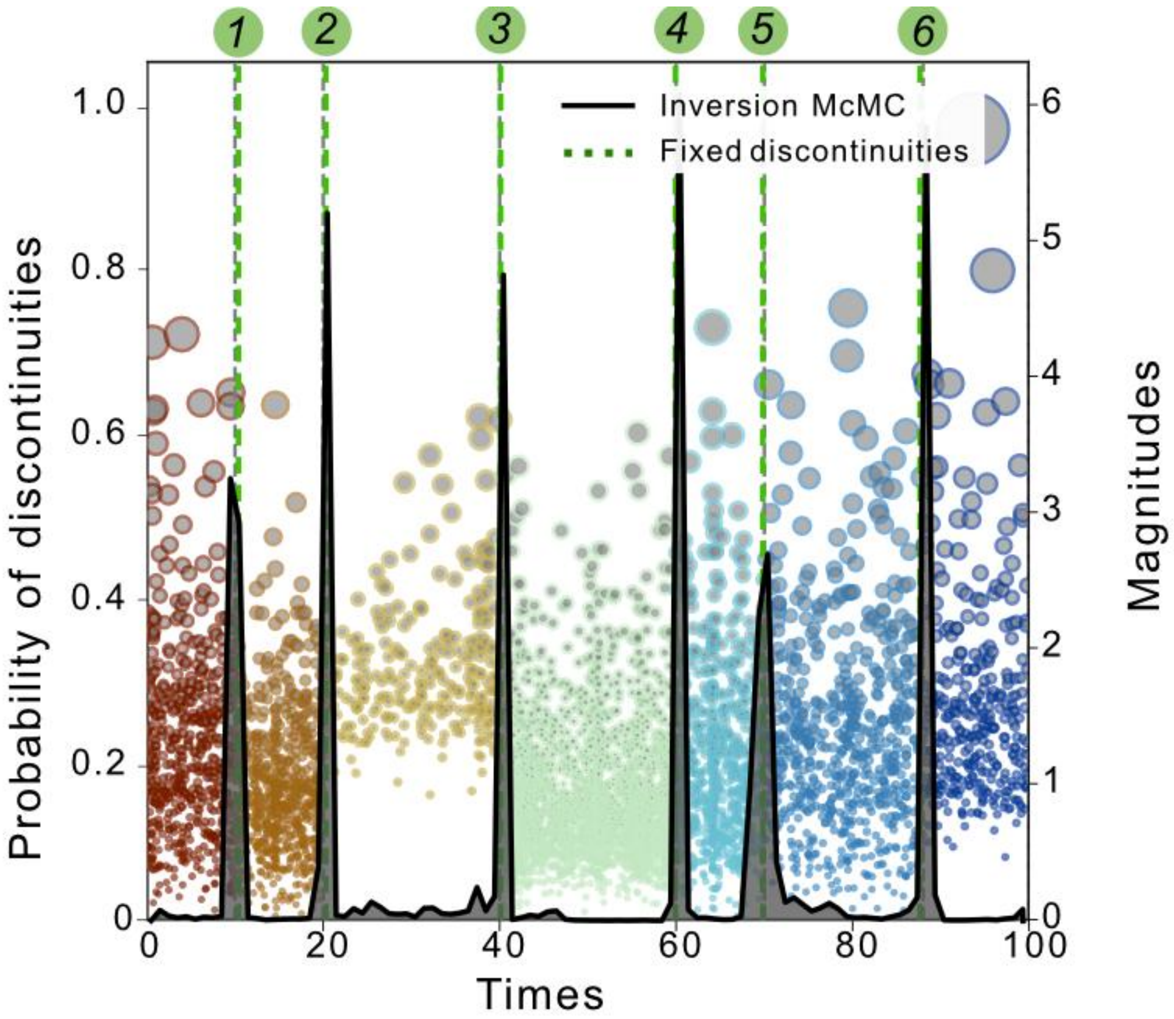


(3) Inversion of temporal changes

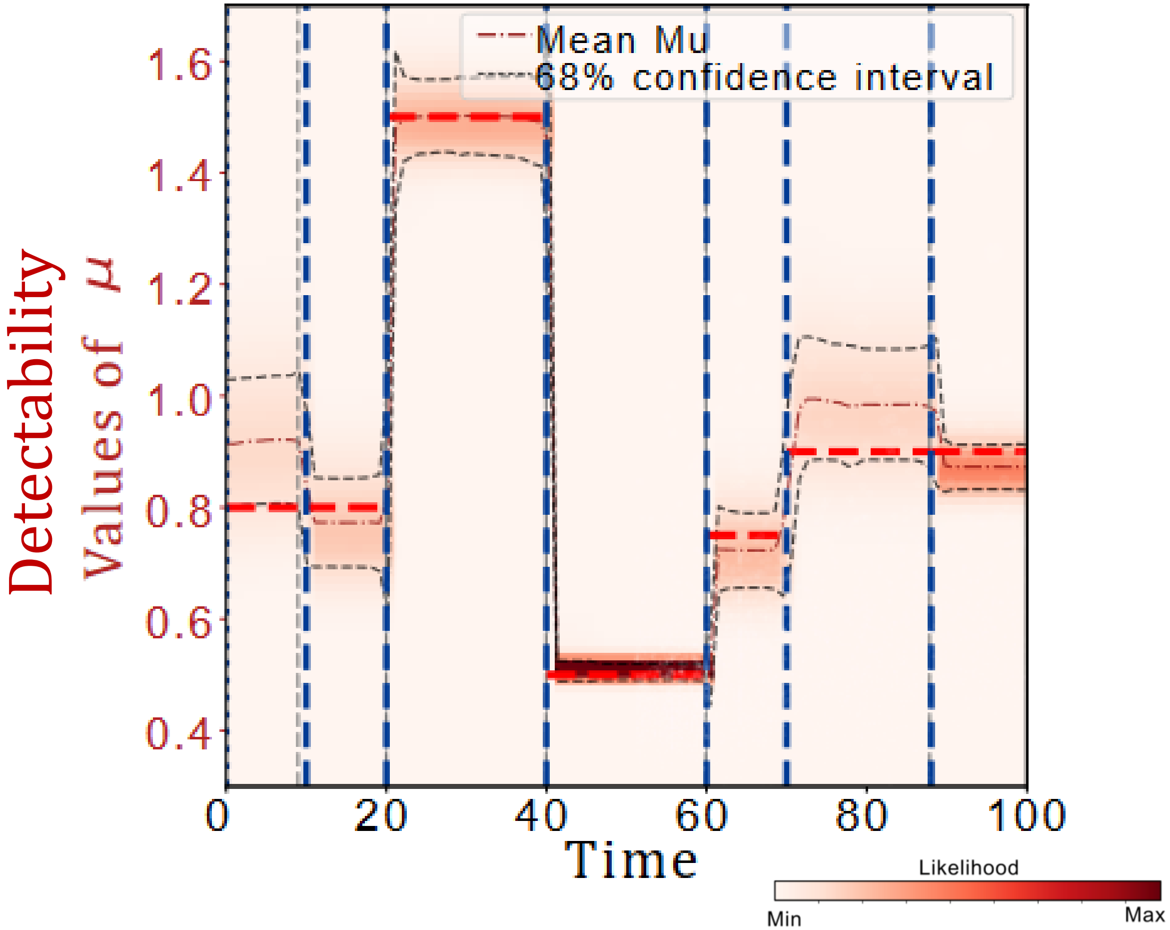


(1) We **retrieve the probability of the temporal discontinuities**

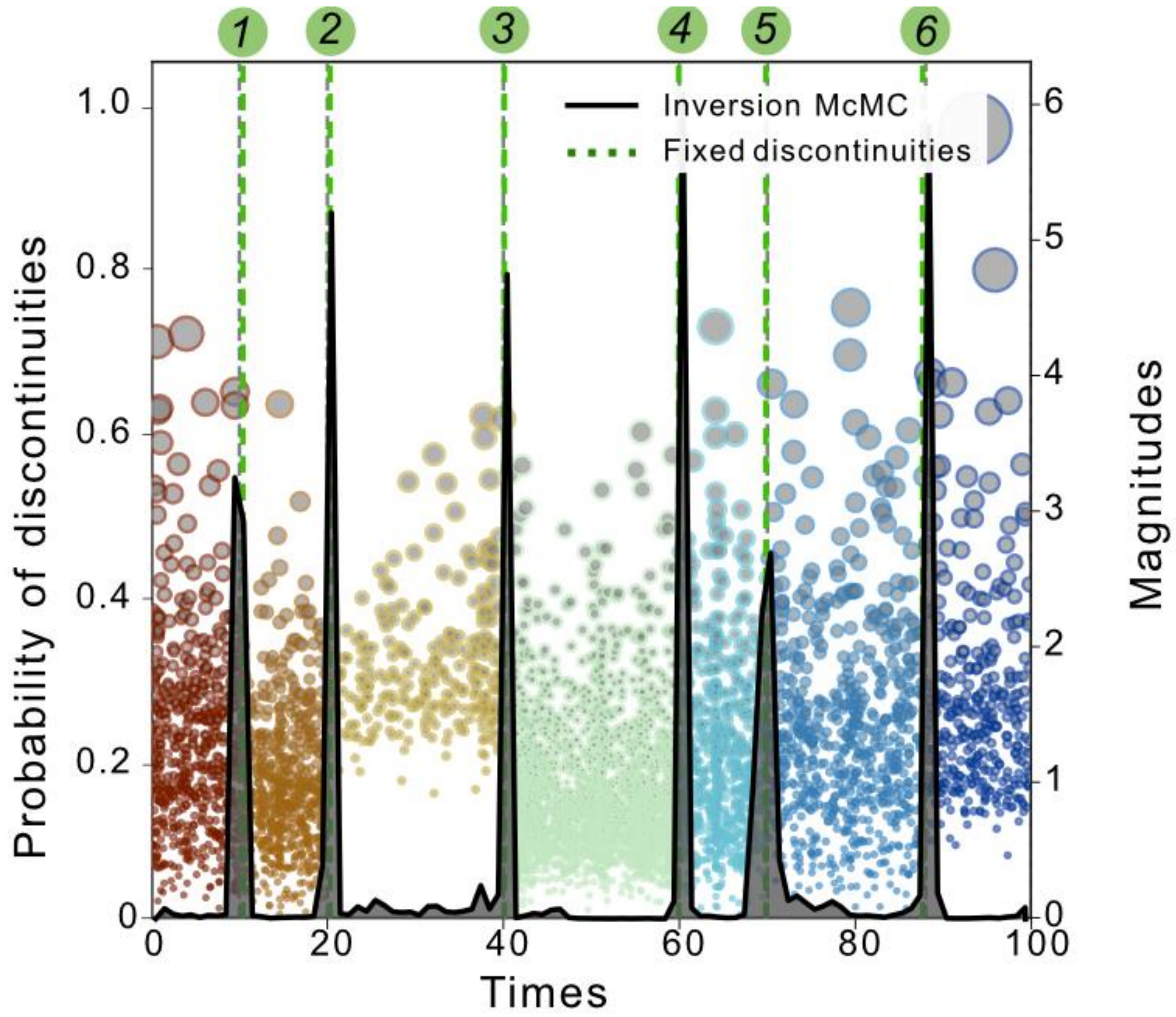
(3) Inversion of temporal changes



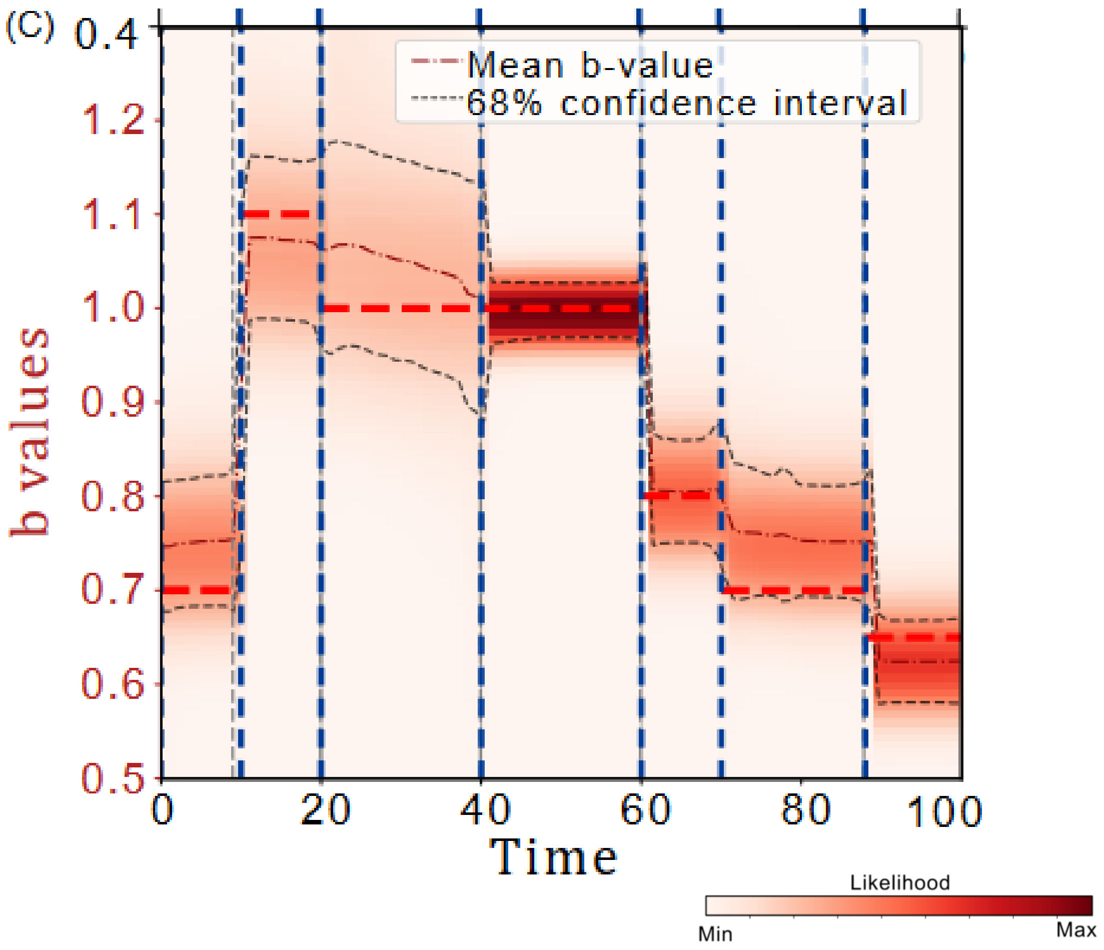
(2) We **get the temporal evolution of the PDF for each parameter of the inversion**



(3) Inversion of temporal changes

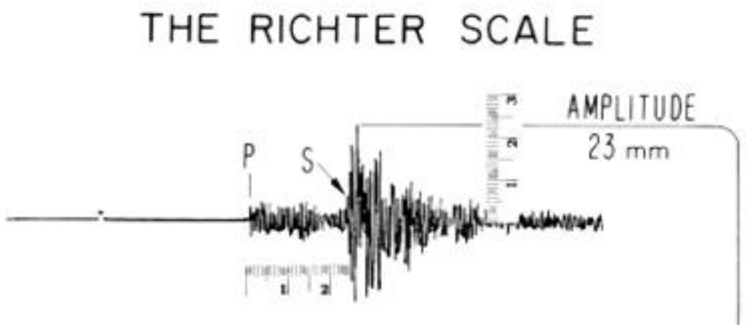


(2) We **get the temporal evolution of the PDF for each parameter of the inversion**

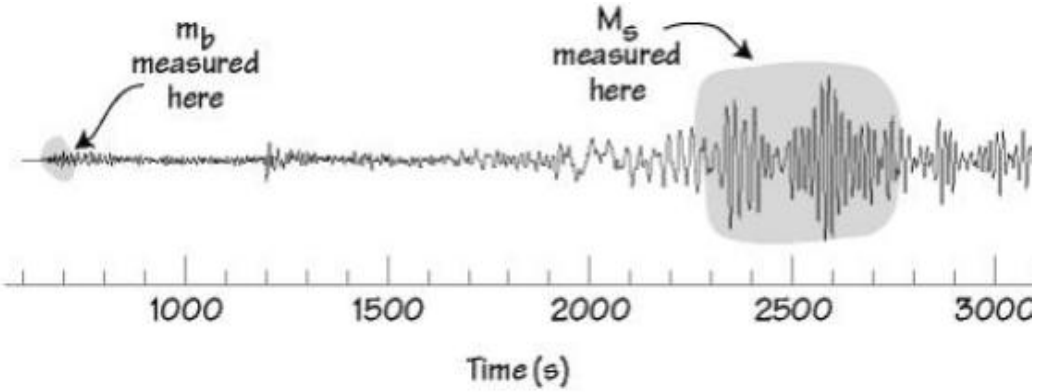


Introduction : what magnitude ?

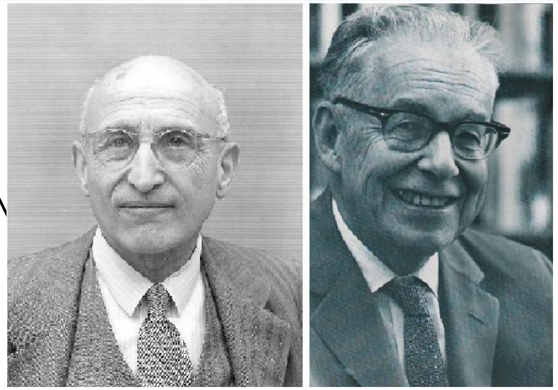
- **Regional/Local**



- **Teleseismic (>3000 km)**



Earthquake magnitudes are governed by seismic **amplitude** and **distance**



Gutenberg and Richter, 1950s

Richter scale (Local magnitude M_L)

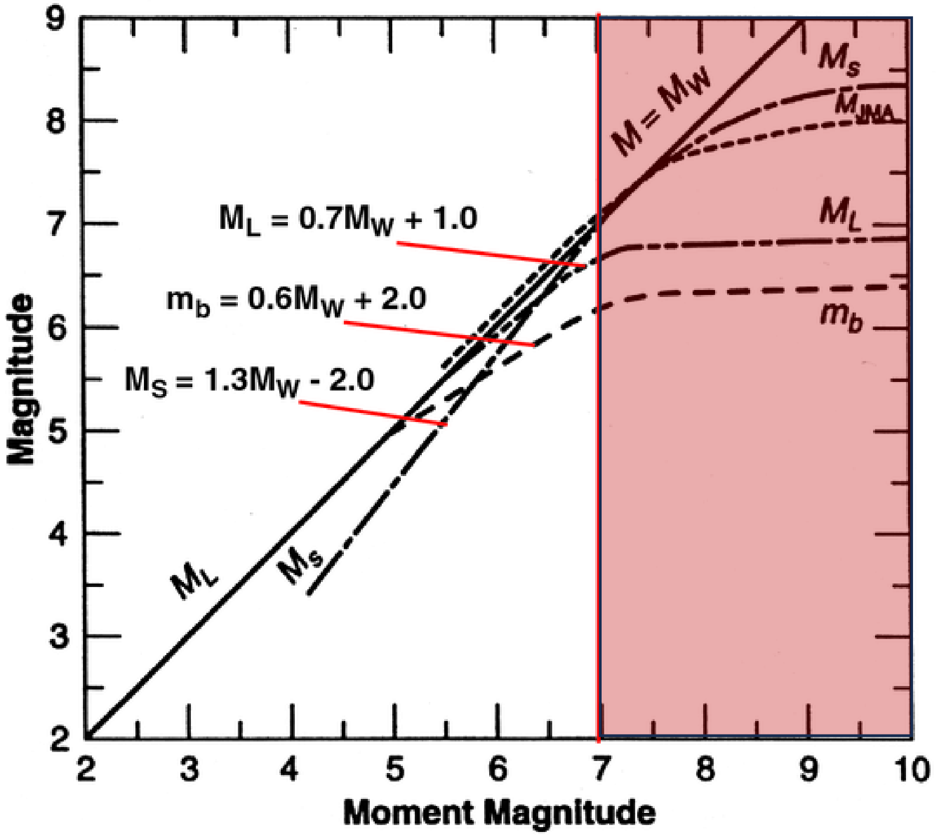
$$M_L = log(Amplitude) + 2.76\ log(Distance) - 2.48$$

From teleseismic observations :

mb *(volumic)* **and Ms** *(surface waves)*

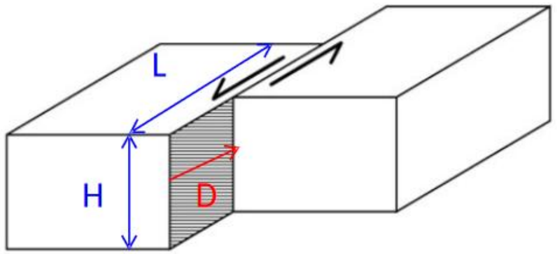
Introduction : what magnitude ?

For earthquakes above magnitude 7 saturation of M_L , m_b , M_S



Moment magnitude Mw *(Hanks and Kanamori)*

$$M_w = \frac{2}{3} log(M_0) - 10.7$$



Seismic moment can be estimated from the estimated surface that breaks during large earthquakes

Magnitude scales are not always equivalent